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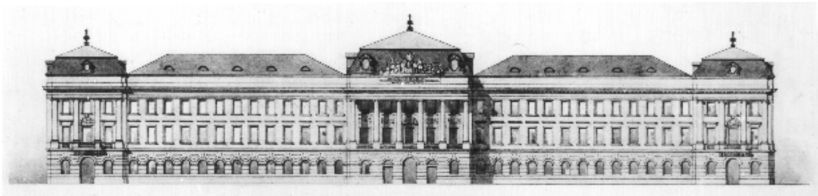

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The Complexity Landscape of Claim-Augmented Argumentation Frameworks

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The Complexity Landscape of Claim-Augmented Argumentation Frameworks

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Abstract. Claim-augmented argumentation frameworks (CAFs) provide a formal basis to analyze conclusion-oriented problems in argumentation by adapting a claim-focused perspective; they extend Dung AFs by associating a claim to each argument representing its conclusion. This additional layer offers various possibilities to generalize abstract argumentation semantics, i.e. the re-interpretation of arguments in terms of their claims can be performed at different stages in the evaluation of the framework: One approach is to perform the evaluation entirely at argument-level before interpreting arguments by their claims (inherited semantics); alternatively, one can perform certain steps in the process (e.g., maximization) already in terms of the arguments' claims (claim-level semantics). The inherent difference of these approaches not only potentially results in different outcomes but, as we will show in this paper, is also mirrored in terms of computational complexity. To this end, we provide a comprehensive complexity analysis of the four main reasoning problems with respect to claim-level variants of preferred, naive, stable, semi-stable and stage semantics and complete the complexity results of inherited semantics by providing corresponding results for semi-stable and stage semantics. Furthermore, we provide complexity results for these types of frameworks when restricted to specific graph classes and when parameterized by the number of claims within the framework. Moreover, we show that deciding, whether for a given framework the two approaches of a semantics coincide (concurrence) can be surprisingly hard, ranging up to the third level of the polynomial hierarchy.

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1 Introduction

Argumentation is an increasingly important research area within AI [1]. Among the most prominent approaches to handle inconsistent and conflicting statements is abstract argumentation [2] which is nowadays acknowledged as one of the core reasoning mechanisms for argumentation. In his seminal paper, Dung has proposed several argumentation semantics which have been adopted subsequently in several formalisms [3, 4]. Over the past decades, many more semantics entered the stage, each of which contributes to the rich and diverse landscape of argumentation semantics [5]. By now, the broad variety of semantics for argumentation offers many choices to model argumentative settings as needed. Despite of all differences, most of the argumentation semantics have something in common: their high computational complexity. Indeed, it has been shown that deciding credulous as well as skeptical acceptance of arguments but also the verification of sets of jointly acceptable arguments is computationally expensive, ranging up to the second level of the polynomial hierarchy [6].

Although a lot of effort has been invested in exploring the computational complexity of the semantics in terms of arguments, only little is known about the complexity of evaluating argumentative settings in terms of the claims of the arguments. Generally speaking, the *claim* of an argument is the statement it intends to justify. Ultimately, an argumentative analysis aims to identify justifiable assertions; hence the evaluation of claim acceptance is an essential part of argumentative reasoning.

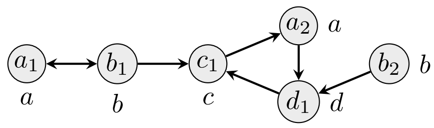
As recently addressed in the literature, there are several ways to transfer argument acceptance to claim acceptance [7, 8]. Let us outline two intuitive approaches in the general schema to instantiate argumentation frameworks, so called *instantiation procedures* (see e.g. [9, 10, 4, 11]). This instantiation process starts from a (typically inconsistent) knowledge base, from which possible arguments are constructed. An argument consists of a claim and a support, the latter being a subset of the knowledge base. The relationship between arguments is then settled, for instance an argument α attacks argument β if the claim of α contradicts (parts of) the support of β. As soon as all arguments and attacks between arguments are given, one abstracts away from the contents of the arguments. The resulting network is then interpreted as an abstract argumentation framework (AF) and semantics for AFs are used to obtain a collection of jointly acceptable sets of arguments, commonly referred to as extensions. One of the most famous argumentation semantics are *preferred semantics* which return maximal admissible (i.e., conflict-free and self-defending) sets of arguments. To obtain the preferred set of claims, these extensions are then reinterpreted in terms of the claims of the accepted arguments, thus restating the result in the domain of the initial setting. We recall two natural choices to obtain our desired preferred claim-sets. When looking for preferred extensions in terms of claims, we can either

(a) take the preferred extensions of the AF and replace each argument by its claim, or

(b) take the admissible sets of the AF, replace each argument by its claim, and then select the subset-maximal ones from the resulting set of extensions.

Option (a) which we shall call *inherited semantics* in what follows, is often used implicitly in instantiation-based argumentation and has been explicitly studied in [12]. This approach resembles reasoning methods in rule-based formalisms such as ASPIC+ [4]. Option (b) has recently been advocated in [8] as an alternative way to lift concepts behind argumentation semantics to claim-based semantics; we will refer to the latter as *claim-level semantics* since parts of the semantic selection process takes place on the claim- rather than on the argument-level. Hence, these two approaches provide different methods in order to accomplish the final steps in the instantiation process, i.e., evaluating the abstract framework and provide the extensions in terms of the accepted claims. Understanding the complexity of this part in the instantiation is crucial towards the design of advanced argumentation systems. Investigating this final step independently from the entire process has the clear advantage that results are not restricted to a particular formalism (e.g., ASPIC+) and are thus of general nature. Furthermore, as discussed in [13], there are logic programming semantics that, in the standard instantiation model [14, 10], correspond to claim-level semantics and cannot be captured with inherited semantics.

Example 1. *Consider the following AF where each node represents an argument and the edges representing their relations, i.e., attacks between them. Each argument is labelled with its respective claim, i.e., arguments a_1 and a_2 are assigned claim a, arguments b_1 and b_2 are assigned claim b and arguments c_1 and d_1 are assigned claims c and d respectively.*



Evaluating the AF with respect to the admissible semantics, ignoring the claims, yields $\emptyset, \{a_1\}, \{b_1\}, \{b_2\}, \{a_1, b_2\}, \{b_1, b_2\}, \{a_2, b_1\}, \{a_1, b_2, c_1\}$*, and* $\{a_2, b_1, b_2\}$*. To obtain the preferred claim-sets one can now select the subset-maximal sets and then replace each argument by its claim (option (a)), yielding* $\{a, b, c\}, \{a, b\}$*; observe that swapping those steps (option (b)) results in the unique claim-set* $\{a, b, c\}$*.*

In [12], it has been shown that inherited semantics are in general of higher computational complexity than their argument-based counterparts. In particular the verification problem is computationally more expensive. While the computational complexity of inherited semantics has already been investigated for many argumentation semantics, the computational complexity of claim-level semantics has not been studied so far. As we already observed in the above example, the two approaches to evaluate the framework with respect to preferred semantics yield different results. A detailed analysis of the differences between these two approaches was provided in [8], also showing that there are some semantics where the two approaches coincide when arguments with the same claim attack the same arguments (this property is commonly referred to as *well-formedness*). What remains open is the question whether this difference is mirrored in terms of computational complexity. In that matter, we are in particular interested in deciding whether these approaches yield the same result in a given framework. Hence apart from the classical decision problems of deciding credulous and skeptical acceptance, verification of acceptance for a given claim set, and deciding whether a non-empty set of acceptable claims exist, we furthermore consider the question of how hard it is to decide whether two different approaches of a semantics deliver the same result. We call this decision problem *concurrence* of two frameworks. As sketched above, there are some situations in which inherited and claim-level semantics yield the same outcome; namely in case the considered argumentation framework satisfies well-formedness which is a certain structural restriction that appears naturally in many instantiation procedures. Tying into this, as many of the obtained results will conclude intractability, considering specific graph classes or parameterized decision problems can be useful. This has been done for AFs [15] and for some inherited semantics [12], but is still an open question for some of the other common semantics that output claim-sets as result of their evaluation.

We tackle these three questions via a thorough complexity analysis. To be independent from a particular instantiation schema, we consider claim-augmented frameworks (CAFs) [12], which are AFs where each argument is assigned a claim (indeed Example 1 provides an example for a CAF).

Our main contributions are as follows:

- We settle the computational complexity of all the claim-level semantics, i.e. stable, naive, preferred, semi-stable, and stage semantics, introduced in [8] for the main decision problems of credulous and skeptical acceptance, verification, and testing for non-empty extensions. Among our findings is that for naive semantics, the claim-level variant is harder than its inherited counterpart, while for preferred semantics, it is the inherited variant that shows higher complexity.

- We also provide complexity results for inherited semi-stable and stage semantics which have not been investigated in [12]. As it turns out, for these two semantics the complexity of the inherited and claim-level variants coincides.

- Additionally, we provide complexity results for the main decision problems when restricted to specific graph classes and also when parameterized by the number of claims for inherited semi-stable and stage semantics as well as for the claim-level variants of the stable, naive, preferred, semi-stable, and stage semantics. As we will see, this will often times allow for better bounds than the unrestricted case.

- We determine the complexity of the concurrence problem, i.e. whether for a given CAF and a semantics, the inherited and claim-level variant of that semantics coincide. Note that showing this problem to be easy would suggest that there are relatively natural classes of CAFs which characterize whether or not the two variants collapse. However, as we will see, concurrence can be surprisingly hard, up to the third level of the polynomial hierarchy.

A preliminary version of this paper has been presented at the thirty-fifth AAAI conference on artificial intelligence (AAAI-21) [16]. Besides providing full proofs and in-depth discussions, this version significantly extends the preceding paper by several new complexity results, in particular, we provide a full complexity analysis of the considered reasoning problems for specific graph classes.

2 Preliminaries

In the this section we (a) recall abstract argumentation frameworks, claim-augmented argumentation frameworks and their semantics, and (b) recall the necessary background and computational complexity,

2.1 Argumentation Frameworks and their Semantics

We introduce (abstract) argumentation frameworks and their semantics [2, 5]. We fix U as countable infinite domain of arguments.

Definition 1. *An* argumentation framework (AF) *is a pair* $F = (A, R)$ *where* $A \subseteq U$ *is a finite set of arguments and* $R \subseteq A \times A$ *is the attack relation.* $E \subseteq A$ attacks b *if* $(a, b) \in R$ *for some* $a \in E$*; we denote by* $E^+_F = \{b \in A \mid \exists a \in E : (a, b) \in R\}$ *the set of arguments defeated by* E*. We call* $E^\oplus_F = E \cup E^+_F$ *the* range *of* E *in* F*. An argument* $a \in A$ *is* defended *(in* F*) by* E *if* $b \in E^+_F$ *for each* b *with* $(b, a) \in R$.

Semantics for AFs are defined as functions σ which assign to each AF $F = (A, R)$ a set $\sigma(F) \subseteq 2^A$ of extensions. We consider for σ the functions *cf*, *adm*, *naive*, *prf*, *stb*, *sem* and *stg* which stand for conflict-free, admissible, naive, preferred, stable, semi-stable and stage, respectively.

Definition 2. *Let $F = (A, R)$ be an AF. A set $E \subseteq A$ is* conflict-free (in F), *if there are no $a, b \in E$, such that $(a, b) \in R$. $cf(F)$ denotes the collection of conflict-free sets in F. For $E \in cf(F)$ we have $E \in adm(F)$ if each $a \in E$ is defended by E in F. For $E \in cf(F)$, we define*

- *$E \in naive(F)$, if there is no $D \in cf(F)$ with $E \subset D$;*
- *$E \in prf(F)$, if $E \in adm(F)$ and $\nexists D \in adm(F)$: $E \subset D$;*
- *$E \in stb(F)$, if $E^{\oplus}_F = A$;*
- *$E \in sem(F)$, if $E \in adm(F)$ and $\nexists D \in adm(F)$: $E^{\oplus}_F \subset D^{\oplus}_F$;*
- *$E \in stg(F)$, if there is no $D \in cf(F)$ with $E^{\oplus}_F \subset D^{\oplus}_F$.*

Next we introduce claim-augmented argumentation frameworks (CAFs) [12], which extend AFs by a function *claim* that assigns claims to argument.

Definition 3. *A* claim-augmented argumentation framework (CAF) *is a triple $(A, R, claim)$ where (A, R) is an AF and $claim : A \to C$ assigns a claim to each argument in A; C is a set of possible claims. The claim-function is extended to sets in the natural way, i.e. $claim(E) = \{claim(a) \mid a \in E\}$. A set of arguments $E \subseteq A$ is called a* realization *of a claim-set $S \subseteq claim(A)$ if $claim(E) = S$. A CAF $(A, R, claim)$ is* well-formed *if $\{a\}^+_{(A,R)} = \{b\}^+_{(A,R)}$ for all $a, b \in A$ with $claim(a) = claim(b)$.*

Well-formed CAFs naturally appear as result of instantiation procedures where the construction of the attack relation depends on the claim of the attacking argument. However, formalisms which handle argument strengths or allow for preference relations over arguments (assumptions/defeasible rules) typically violate the property of well-formedness [17, 18].

Semantics for CAFs Here we give a short recap of *inherited semantics* and *claim-level semantics* for CAFs. We will first introduce inherited semantics (i-semantics).

Definition 4. *For a CAF $CF = (A, R, claim)$ and an AF semantics σ, we define* i-σ semantics *as $\sigma_c(CF) = \{claim(E) \mid E \in \sigma((A, R))\}$. We call $E \in \sigma((A, R))$ with $claim(E) = S$ a σ_c-realization of S in CF.*

Next we discuss claim-level semantics (cl-semantics) for CAFs. Central for cl-variants of stable, semi-stable and stage semantics is the following notion of claim-defeat.

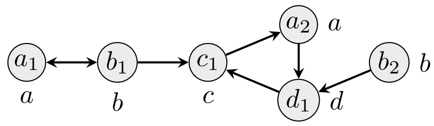
Definition 5. *Let $CF = (A, R, claim)$, $E \subseteq A$ and $c \in claim(A)$. E* defeats c (in CF) *if E attacks (in (A, R)) every $a \in A$ with $claim(a) = c$.*

Example 2. *Consider the CAF from Example 1. The argument b_1 (or: the set $E = \{b_1\}$) attacks the arguments a_1, c_1 but defeats only claim c, because not every occurrence of claim a is attacked.*

We will next introduce the notion of range for a claim-set S. As different realizations of S might yield different sets of defeated claims, the range of S is in general not unique and depends on the particular realization E of S.

Definition 6. *For a CAF $CF = (A, R, claim)$, let $\nu_{CF}(E) = \{c \in claim(A) \mid E$ defeats c in $CF\}$. For a claim-set $S \subseteq claim(A)$ and a realization E of S in CF, we call $S \cup \nu_{CF}(E)$ a* range *of S in CF. If $S \cup \nu_{CF}(E) = claim(A)$ we say E has full claim-range.*

Example 3. *Let us consider again the CAF CF from Example 1.*



First, consider the set of arguments $E_1 = \{b_2, c_1\}$. The set attacks the arguments d_1 and a_2. Hence E_1 attacks claim d; claim a however is not attacked since E_1 does not attack all occurrences of a (the argument a_1 is unattacked). Thus $\nu_{CF}(E_1) = \{d\}$ and $claim(E_1) \cup \nu_{CF}(E_1) = \{b, c, d\}$. Now, we extend E_1 by a_1 and obtain $E_2 = \{a_1, b_2, c_1\}$. Again, claim d is the only claim which is attacked by the set; however, E_2 has full claim-range since it contains claim a.

Observe that in well-formed CAFs, each claim-set possesses a unique range as each realization attacks the same arguments, i.e., for a claim-set $S \subseteq claim(A)$, $\nu_{CF}(E) = \nu_{CF}(D)$ for all realizations E, D of S in CF. We will thus write S^+_{CF} to denote the unique set of defeated claims $\nu_{CF}(E)$ of S in CF.

We are now ready to introduce cl-semantics for CAFs.

Definition 7. *For a CAF $CF = (A, R, claim)$ and $S \subseteq claim(A)$, we define*

- *$S \in cl\text{-}prf(CF)$ if $S \in adm_c(CF)$ and there is no $T \in adm_c(CF)$ with $S \subset T$;*
- *$S \in cl\text{-}naive(CF)$ if $S \in cf_c(CF)$ and there is no $T \in cf_c(CF)$ with $S \subset T$;*
- *$S \in cl\text{-}stb_\tau(CF)$, $\tau \in \{cf, adm\}$, if there exists $E \in \tau((A, R))$ with $claim(E) = S$ and $S \cup \nu_{CF}(E) = claim(A)$;*
- *$S \in cl\text{-}sem(CF)$ if there exists $E \in adm((A, R))$ with $claim(E) = S$ such that there is no $D \in adm((A, R))$ with $S \cup \nu_{CF}(E) \subset claim(D) \cup \nu_{CF}(D)$;*
- *$S \in cl\text{-}stg(CF)$ if if there exists $E \in cf((A, R))$ with $claim(E) = S$ such that there is no $D \in cf((A, R))$ with $S \cup \nu_{CF}(E) \subset claim(D) \cup \nu_{CF}(D)$.*

A set of arguments $E \subseteq A$ is a

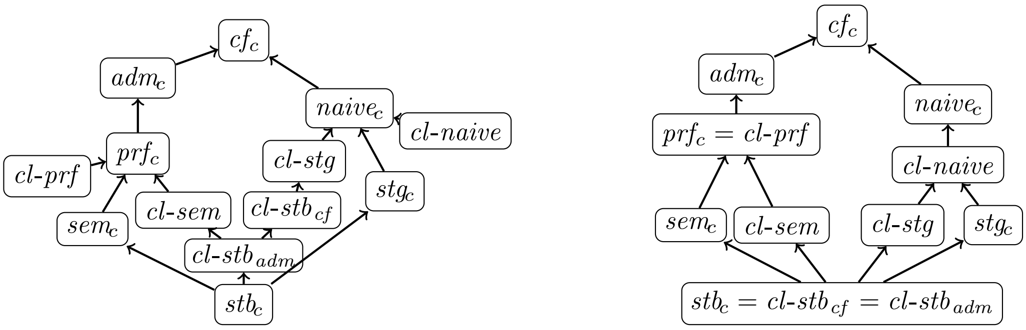
- *cl-prf*-realization *(cl-naive*-realization*) of $S \subseteq claim(A)$ in CF if $claim(E) = S$, $E \in adm((A, R))$ ($E \in cf((A, R))$, respectively);*
- *cl-stb$_\tau$*-realization *of $S \subseteq claim(A)$ in CF, $\tau \in \{adm, cf\}$, if $claim(E) = S$, $E \in adm((A, R))$ ($E \in cf((A, R))$), and $S \cup \nu_{CF}(E) = claim(A)$;*
- *cl-sem*-realization *(cl-stg*-realization*) of $S \subseteq claim(A)$ in CF if $claim(E) = S$, $E \in adm((A, R))$ ($E \in cf((A, R))$), and $S \cup \nu_{CF}(E)$ is subset-maximal among admissible respectively conflict-free range-sets in CF.*

Example 4. *Let us consider again our running example CAF CF. As we have observed already in Example 1, the preferred claim-sets of CF are given by $\{a, b, c\}$ and $\{a, b\}$. The set E_2 from the above example is cl-stb$_\tau$ (for $\tau \in \{cf, adm\}$) in CF since it has full claim-range. Observe that E_2 is also stable on argument-level since it attacks all other arguments.*

Let $E_3 = \{b_1, a_2\}$. The set is conflict-free, admisssible, and attacks the arguments c_1 and d_1. Hence $claim(E_3) \cup \nu_{CF}(E_3) = claim(A)$, i.e., E_3 has full claim-range and is cl-stb$_\tau$ for $\tau \in \{cf, adm\}$.

We occasionally make use of the relations between different semantics for CAFs [12, 8]. For inherited semantics, the relations between the semantics carry over from the corresponding AF counterparts, e.g.,

$$stb_c(CF) \subseteq sem_c(CF) \subseteq prf_c(CF) \subseteq adm_c(CF)$$



(a) Relations for CAFs. (b) Relations for well-formed CAFs.

Figure 1: Relations between semantics for general CAFs (a) and well-formed CAFs (b) as presented in [8]. An arrow from σ to τ indicates that $\sigma(CF) \subseteq \tau(CF)$ for each CAF CF.

for any CAF CF. The relations between the different variants for the semantics often depend on the particular CAF class, e.g., for general CAFs,

$$stb_c(CF) \subseteq \mathit{cl\text{-}stb}_{adm}(CF) \subseteq \mathit{cl\text{-}stb}_{cf}(CF).$$

For well-formed CAFs, on the other hand, all stable variants coincide, i.e., $stb_c(CF) = \mathit{cl\text{-}stb}_{adm}(CF) = \mathit{cl\text{-}stb}_{cf}(CF)$. Figure 1 provides an overview over the relations between semantics for general and for well-formed CAFs. We furthermore observe the following implications between claim-level stable semantics and semi-stable respectively stage semantics: If $\mathit{cl\text{-}stb}_{cf}(CF) \neq \emptyset$ then $\mathit{cl\text{-}stb}_{cf}(CF) = \mathit{cl\text{-}stg}(CF)$, likewise, if $\mathit{cl\text{-}stb}_{adm}(CF) \neq \emptyset$ then $\mathit{cl\text{-}stb}_{adm}(CF) = \mathit{cl\text{-}sem}(CF)$ for each CAF CF [13].

Remark 1. *Let us briefly discuss why we do not consider claim-level versions for complete, grounded, and admissible semantics. An appropriate adaption of both semantics requires a notion for claim-defense. As discussed in [19], the natural choice of adapting a defense notion to claim-level (a claim c is defended by a set of arguments E iff there exists some occurrence of c which is defended by E) results in cl-complete, cl-grounded, and cl-admissible semantics that are equivalent to their inherited counter-parts.*

2.2 Computational Complexity

We assume the reader to be familiar with the basic concepts of computational complexity theory (see, e.g. [20] for an introduction), in particular with the complexity classes polynomial time (P)and non-deterministic polynomial time (NP). In the following, we briefly recapitulate the concept of oracle machines and related complexity classes relevant for this work. To this end, let C denote some complexity class. By a C-oracle machine we mean a (polynomial time) Turing machine which can access an oracle that decides a given (sub)-problem in C within one computation step. We denote the corresponding complexity classes of such machines as P^{C} if the underlying Turing machine is deterministic; and NP^{C} if the underlying Turing machine is nondeterministic. In this work we consider complexity classes from the first three levels of the polynomial-time hierarchy. The classes NP and coNP build the first level of the polynomial-time hierarchy. The complexity classes on the second level are build by the use of NP-oracles. First, the class $\Sigma_2^{\mathsf{P}} = \mathsf{NP}^{\mathsf{NP}}$ denotes the set of problems which can be decided by a nondeterministic polynomial time algorithm that has (unrestricted) access to an NP-oracle. The class $\Pi_2^{\mathsf{P}} = \mathsf{coNP}^{\mathsf{NP}}$ is defined as the complementary class of Σ_2^{P}, i.e. $\Pi_2^{\mathsf{P}} = \mathsf{co}\Sigma_2^{\mathsf{P}}$. In the same way we can define the third level by using Σ_2^{P} oracles. That is, we define the class Σ_3^{P} as $\mathsf{NP}^{\Sigma_2^{\mathsf{P}}}$ and $\Pi_3^{\mathsf{P}} = \mathsf{coNP}^{\Sigma_2^{\mathsf{P}}}$ as the complementary class of Σ_3^{P}.

We have the following relations between these complexity classes:

$$\mathsf{P} \subseteq \begin{matrix} \mathsf{NP} \\ \mathsf{coNP} \end{matrix} \subseteq \begin{matrix} \Sigma_2^\mathsf{P} \\ \Pi_2^\mathsf{P} \end{matrix} \subseteq \begin{matrix} \Sigma_3^\mathsf{P} \\ \Pi_3^\mathsf{P} \end{matrix}$$

We will see that many problems in this paper are indeed of high complexity. A prominent approach to tame the high complexity of such problems is parameterized complexity theory (see, e.g., [21]). A key observation of this approach is that many hard problems become polynomial-time tractable if some problem parameter is bounded by a fixed constant. If the order of the polynomial bound is independent of the parameter[1] one speaks of *fixed-parameter tractability* (FPT).

3 Computational Problems

We consider the following decision problems with respect to a CAF-semantics σ:

- *Credulous Acceptance* ($\mathit{Cred}_\sigma^{CAF}$): Given a CAF $CF = (A, R, \mathit{claim})$ and claim $c \in \mathit{claim}(A)$, is c contained in some $S \in \sigma(CF)$?
- *Skeptical Acceptance* ($\mathit{Skept}_\sigma^{CAF}$): Given a CAF $CF = (A, R, \mathit{claim})$ and claim $c \in \mathit{claim}(A)$, is c contained in each $S \in \sigma(CF)$?
- *Verification* ($\mathit{Ver}_\sigma^{CAF}$): Given a CAF $CF = (A, R, \mathit{claim})$ and a set $S \subseteq \mathit{claim}(A)$, is $S \in \sigma(CF)$?
- *Non-emptiness* (NE_σ^{CAF}): Given a CAF $CF = (A, R, \mathit{claim})$, is there a non-empty set $S \subseteq \mathit{claim}(A)$ such that $S \in \sigma(CF)$?

We furthermore consider these reasoning problems restricted to well-formed CAFs and denote them by $\mathit{Cred}_\sigma^{wf}$, $\mathit{Skept}_\sigma^{wf}$, Ver_σ^{wf}, and NE_σ^{wf}. Moreover, we denote the corresponding decision problems for AFs (which can be obtained by defining *claim* as the identity function) by $\mathit{Cred}_\sigma^{AF}$, $\mathit{Skept}_\sigma^{AF}$, Ver_σ^{AF}, and NE_σ^{AF}. Finally, we introduce a new decision problem which asks whether the two variants of a semantics coincide on a given CAF.

- *Concurrence* ($\mathit{Con}_\sigma^{CAF}$): Given a CAF CF, does it hold that $\sigma_c(CF) = \mathit{cl}\text{-}\sigma(CF)$?

For stable semantics, we write $\mathit{Con}_{stb_\tau}^{CAF}$ to specify the considered cl-stable variant ($\tau \in \{\mathit{adm}, \mathit{cf}\}$). The concurrence problem restricted to well-formed CAFs is denoted Con_σ^{wf}.

Tables 1 & 2 depict known complexity results for AF semantics [22, 23, 24, 6]; and for inherited CAF semantics [12]. Note that Table 2 lacks results for semi-stable and stage semantics which have not been studied yet in terms of complexity. We close this gap and complement these results by an analysis of the claim-level variants.

4 Complexity of Reasoning Problems

The forthcoming analysis yields the following high level picture: Credulous and skeptical reasoning as well as deciding existence of a non-empty extension is of the same complexity as in AFs except for the notable difference that skeptical reasoning with respect to cl-naive semantics goes up two levels in the polynomial hierarchy and is thus also more complex

[1] That is, the running time can be stated as $O(f(k) \cdot \mathit{poly}(n))$, where f is a computable function, k is the problem parameter under investigation, n is the size of the problem instance, and $\mathit{poly}(\cdot)$ is an arbitrary but fixed polynomial.

Table 1: Complexity of AFs.

σ	$Cred_\sigma^{AF}$	$Skept_\sigma^{AF}$	Ver_σ^{AF}	NE_σ^{AF}
cf	in P	trivial	in P	in P
adm	NP-c	trivial	in P	NP-c
stb	NP-c	coNP-c	in P	NP-c
$naive$	in P	in P	in P	in P
prf	NP-c	Π_2^P-c	coNP-c	NP-c
sem	Σ_2^P-c	Π_2^P-c	coNP-c	NP-c
stg	Σ_2^P-c	Π_2^P-c	coNP-c	in P

Table 2: Known complexity results for inherited semantics, with $\Delta \in \{CAF, wf\}$. Results that deviate from the corresponding results for AFs are bold-face.

σ	$Cred_\sigma^{\Delta}$	$Skept_\sigma^{\Delta}$	Ver_σ^{CAF} / Ver_σ^{wf}	NE_σ^{Δ}
cf_c	in P	trivial	**NP**-c / in P	in P
adm_c	NP-c	trivial	**NP**-c / in P	NP-c
stb_c	NP-c	coNP-c	**NP**-c / in P	NP-c
$naive_c$	in P	**coNP**-c	**NP**-c / in P	in P
prf_c	NP-c	Π_2^P-c	$\boldsymbol{\Sigma_2^P}$-c / coNP-c	NP-c
sem_c	?	?	? / ?	?
stg_c	?	?	? / ?	?

than deciding skeptical acceptance for i-naive semantics which has been shown to be coNP-complete. For well-formed CAFs, skeptical reasoning admits the same complexity for both claim-level and inherited naive semantics but remains more complex than in AFs.

For general CAFs, the verification problem is more complex than for AFs for all of the considered semantics. Comparing claim-level and inherited semantics we observe that the complexity of the verification problem for cl-preferred semantics drops while the complexity for cl-naive semantics admits a higher complexity than their inherited counterparts; the claim-level and inherited variants of stable, semi-stable and stage semantics admit the same complexity. For well-formed CAFs, the complexity of the verification problem coincides with the known results for AFs.

4.1 Complexity Results for General CAFs

In this section, we provide complexity results for general CAFs for credulous and skeptical acceptance, verification and for the non-emptiness problem with respect to both variants of semi-stable and stage semantics as well as claim-level naive, preferred and stable semantics. First, we discuss upper bounds in Section 4.1.1 before we present hardness results yielding the corresponding lower bounds in Section 4.1.2. An overview of our results is given in Tables 3 & 4.

4.1.1 Membership Results

We will first discuss the membership proofs of the considered decision problems. To begin with, we will give poly-time respectively coNP procedures for deciding whether a given set of arguments E is a σ-realization for $\sigma \in \{cl\text{-}stb_{adm}, cl\text{-}stb_{cf}, cl\text{-}sem, cl\text{-}stg\}$. This lemma yields upper bounds for the respective reasoning problems; notice that the complexity goes up one level in the polynomial hierarchy since one requires an additional guess for E.

Lemma 1. *Given a CAF $CF = (A, R, claim)$ and some $E \subseteq A$. Deciding whether E realizes (1) a τ-cl-stable claim-set in CF for $\tau \in \{adm, cf\}$ is in* P*; (2) a cl-semi-stable (cl-stage) claim set in CF is in* coNP*.*

Proof. Checking admissibility (conflict-freeness) of E is in P (cf. Table 1); moreover, $\nu_{CF}(E)$ can be computed in polynomial time by looping over all claims $c \in claim(A)$ and adding each c to $\nu_{CF}(E)$ if E attacks each occurrence of c in CF. For τ-cl-stable semantics, it remains to check whether $claim(E) \cup \nu_{CF}(E) = claim(A)$ to verify that E realizes a τ-cl-stable claim-set in CF. For cl-semi-stable (cl-stage) semantics, we have to check that each $E' \subseteq A$ with $claim(E') \cup \nu_{CF}(E') \supset claim(E) \cup \nu_{CF}(E)$ is not admissible (conflict-free). This can be solved in coNP by a standard guess & check algorithm, i.e. guess a set and verify that it is admissible (conflict-free), compute the claims and verify that they are a proper superset of the claims of the original set, yielding a coNP algorithm to verify that E realizes a cl-semi-stable (cl-stage) claim-set in CF. □

We use this lemma to show membership results for the verification problems for the claim-based semantics.

Proposition 1. *The following membership results hold for the verification problems Ver_σ^{CAF}:*

1. *Ver_σ^{CAF} is in* NP *for $\sigma \in \{cl\text{-}stb_{adm}, cl\text{-}stb_{cf}\}$,*
2. *Ver_σ^{CAF} is in Σ_2^P for $\sigma \in \{cl\text{-}sem, cl\text{-}stg\}$,*
3. *Ver_σ^{CAF} is in* DP *for $\sigma \in \{cl\text{-}prf, cl\text{-}naive\}$.*

Proof. Consider a CAF $CF = (A, R, claim)$ and a set $S \subseteq claim(A)$ that has to be verified against a semantics σ. 1 & 2) Here we can apply a guess and check algorithm. That is, one can verify $S \in \sigma(CF)$ by guessing a set of arguments $E \subseteq A$ with $claim(E) = S$ and checking whether E is a σ-realization of S. The latter is in P, respectively coNP by Lemma 1, yielding NP- and Σ_2^P-procedures for the respective semantics.

3) DP-membership of Ver_σ^{CAF} for $\sigma \in \{cl\text{-}prf, cl\text{-}naive\}$ is by (a) checking that a given claim-set S is admissible (conflict-free) and (b) verifying subset-maximality of S. The former has been shown to be NP-complete (cf. Table 2); the latter is in coNP: Guess a set of arguments E such that $S \subset claim(E)$ and check admissibility (conflict-freeness) of E. Thus Ver_σ^{CAF} can be represented as the intersection of a NP-complete problem and a problem in coNP and lies therefore in DP. □

Next we consider the verification problem for the inherited semantics sem_c and stg_c.

Proposition 2. *Ver_σ^{CAF} is in Σ_2^P for $\sigma \in \{sem_c, stg_c\}$.*

Proof. Σ_2^P-membership of $Ver_{\sigma_c}^{CAF}$ for $\sigma \in \{sem, stg\}$ is by guessing a set $E \subseteq A$ with $claim(E) = S$ and checking $E \in \sigma((A, R))$. The latter is coNP-complete by known results for AFs (cf. Table 1). □

We next turn the reasoning problems, starting with the skeptical acceptance problem $Skept_\sigma^{CAF}$.

Proposition 3. *The following membership results hold for the skeptical acceptance problems $Skept_\sigma^{CAF}$:*

1. *$Skept_\sigma^{CAF}$ is in* coNP *for $\sigma \in \{cl\text{-}stb_{adm}, cl\text{-}stb_{cf}\}$,*

2. $Skept_{\sigma}^{CAF}$ *is in* Π_2^{P} *for* $\sigma \in \{$*cl-prf*, *cl-naive*, *cl-sem*, *cl-stg*$\}$.

3. $Skept_{\sigma}^{CAF}$ *is in* Π_2^{P} *for* $\sigma \in \{sem_c, stg_c\}$.

Proof. Membership proofs for $Skept_{\sigma}^{CAF}$ are by standard guess-and-check algorithms for the complementary problem: For a CAF $CF = (A, R, claim)$ and claim $c \in claim(A)$, guess a set $E \subseteq A$ such that $c \notin claim(E)$ and check $claim(E) \in \sigma(CF)$. 1) For $\sigma \in \{cl\text{-}stb_{\tau}\}$ the latter can be verified in P by Lemma 1, which yields coNP-membership; 2) By the same lemma, that test for $sigma \in \{cl\text{-}sem, cl\text{-}stg\}$, is coNP, thus showing Π_2^{P}-membership; for $\sigma \in \{cl\text{-}prf, cl\text{-}naive\}$, we use the result for Ver_{σ}^{CAF}, i.e., $claim(E) \in \sigma(CF)$ can be verified via two NP-oracle calls, which shows that $Skept_{\sigma}^{CAF}$ is in Π_2^{P}; 3) for $\sigma \in \{sem_c, stg_c\}$, it suffices to check $E \in sem((A, R))$ or $E \in stg((A, R))$–both are in coNP (cf. Table 1)–to derive the desired upper bound. □

Proposition 4. *The following membership results hold for the credulous acceptance problems* $Cred_{\sigma}^{CAF}$:

1. $Cred_{\sigma}^{CAF}$ *is in* P *for* $\sigma \in \{$*cl-naive*$\}$,

2. $Cred_{\sigma}^{CAF}$ *is in* NP *for* $\sigma \in \{cl\text{-}stb_{adm}, cl\text{-}stb_{cf}, cl\text{-}prf\}$.

3. $Cred_{\sigma}^{CAF}$ *is in* Σ_2^{P} *for* $\sigma \in \{$*cl-sem*, *cl-stg*$\}$.

Proof. Membership for $Cred_{\sigma}^{CAF}$ and $\sigma \in \{cl\text{-}stb_{\tau}, cl\text{-}sem, cl\text{-}stg, sem_c, stg_c\}$ are by standard guess-and-check-algorithms: For a CAF $CF = (A, R, claim)$ and claim $c \in claim(A)$, guess a set $E \subseteq A$ such that $c \in claim(E)$ and check $claim(E) \in \sigma(CF)$. For cl-preferred and cl-naive semantics, we exploit the fact a claim $c \in claim(A)$ is credulously accepted with respect to cl-preferred (cl-naive) semantics iff it is contained in some i-admissible (i-conflict-free) claim-set and thus the complexity of $Cred_{\theta}^{CAF}$ for $\theta \in \{cf_c, adm_c\}$ (cf. Table 2) applies. □

Proposition 5. *The following membership results hold for the non-empty problems* NE_{σ}^{CAF}:

1. NE_{σ}^{CAF} *is in* P *for* $\sigma \in \{$*cl-naive*, *cl-stg*$\}$;

2. NE_{σ}^{CAF} *is in* NP *for* $\sigma \in \{cl\text{-}stb_{adm}, cl\text{-}stb_{cf}, cl\text{-}prf, cl\text{-}sem\}$;

3. $NE_{stg_c}^{CAF}$ *is in* P *and* $NE_{sem_c}^{CAF}$ *is in* NP.

Proof. NE_{σ}^{CAF} for $\sigma \in \{sem_c, stg_c, cl\text{-}prf, cl\text{-}naive, cl\text{-}sem, cl\text{-}stg\}$ can be reduced to the respective problem for AFs: for cl-preferred (cl-naive) semantics and both variants of semi-stable (stage) semantics, we have that a CAF has a non-empty claim-set iff a non-empty admissible (conflict-free) set of argument exists, i.e., NE_{σ}^{CAF} $\sigma \in \{cl\text{-}prf, cl\text{-}sem, sem_c, cl\text{-}naive, cl\text{-}stg, stg_c\}$, coincides with either NE_{adm}^{AF} or NE_{cf}^{AF} and we get the complexity directly from Table 1. For $\sigma \in \{cl\text{-}stb_{adm}, cl\text{-}stb_{cf}\}$, NE_{σ}^{CAF} can be verified by guessing a non-empty set $E \subseteq A$ and utilizing Lemma 1 (1) for checking that $claim(E)$ is a τ-cl-stable claim-set of CF. □

4.1.2 Hardness Results

We now turn to the hardness results for the considered decision problems. First observe that one can reduce AF decision problems to the corresponding problems for CAFs by assigning each argument a unique claim. Thus CAF decision problems generalize the corresponding problems for AFs and are therefore at least as hard. It remains to provide hardness proofs for the decision problems with higher complexity. By comparing Table 1 with the membership

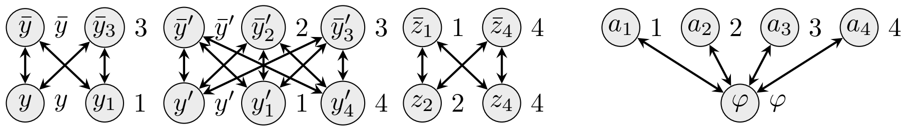


Figure 2: CAF from the proof of Proposition 6 for the formula $\forall yy'\exists z\varphi$, where φ is given by the clauses $\{\{y, y', \neg z\}, \{\neg y', z\}, \{\neg y, \neg y'\}, \{y', z, \neg z\}\}$.

results from above, we observe that it remains to show hardness for $\mathit{Skept}^{CAF}_{\mathit{cl\text{-}naive}}$ and the verification problems $\mathit{Ver}^{CAF}_{\sigma}$ for all semantics σ under consideration.

We will first present a reduction from $QSAT_2^{\forall}$ to show Π_2^{P}-hardness of $\mathit{Skept}^{CAF}_{\mathit{cl\text{-}naive}}$ before we address the verification problems. In this reduction, starting from a QBF $\Psi = \forall Y \exists Z \varphi(Y, Z)$ where φ is a 3-CNF given by a set of clauses $C = \{cl_1, \dots, cl_n\}$ over atoms in $X = Y \cup Z$, we construct a CAF as follows (cf. Figure 2):

- For each clause cl_i, we introduce three arguments representing the literals contained in cl_i and assign them claim i;
- moreover, we add arguments representing literals over Y and assign them unique claims;
- furthermore, we add arguments $a_1, \dots, a_n$ with claims $1, \dots, n$ and an argument φ with unique claim φ;
- we introduce conflicts between each argument representing a variable $x \in X$ and arguments representing its negation; moreover, we add symmetric attacks between φ and each argument a_i.

This reduction is formalized as follows:

Reduction 1. *Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT_2^{\forall}$, where φ is a 3-CNF given by a set of clauses $C = \{cl_1, \dots, cl_n\}$ over atoms in $X = Y \cup Z$. We construct a CAF $CF = (A, R, claim)$ as follows (cf. Figure 2):*

$$\begin{aligned} A = {} & \{x_i \mid x \in cl_i, i \leq n\} \cup \{\bar{x}_i \mid \neg x \in cl_i, i \leq n\} \cup \\ & Y \cup \bar{Y} \cup \{a_1, \dots, a_n, \varphi\} \\ R = {} & \{(a_i, \varphi), (\varphi, a_i) \mid i \leq n\} \cup \{(x_i, \bar{x}_j)(\bar{x}_j, x_i), \mid i, j \leq n\} \cup \\ & \{(y, \bar{y}_i), (\bar{y}_i, y), (y_i, \bar{y}), (\bar{y}, y_i), (y, \bar{y}), (\bar{y}, y) \mid y \in Y\} \end{aligned}$$

where $\bar{Y} = \{\bar{y} \mid y \in Y\}$, and $claim(x_i) = claim(\bar{x}_i) = claim(a_i) = i$, $claim(y) = y$, $claim(\bar{y}) = \bar{y}$, and $claim(\varphi) = \varphi$.

We will show that Ψ is valid iff the claim φ is skeptically accepted with respect to cl-naive semantics in CF. The main observation is that for every $Y' \subseteq Y$, the set $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{a_1, \dots, a_n\}$ is conflict-free in (A, R) by construction, and therefore $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1, \dots, n\}$ is in $cf_c(CF)$. Consequently, φ is skeptically accepted with respect to cl-naive semantics iff for every $Y' \subseteq Y$, the set $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1, \dots, n, \varphi\}$ is cl-naive. It suffices to check that for every $Y' \subseteq Y$, the set $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1, \dots, n, \varphi\}$ is cl-naive iff there is $Z' \subseteq Z$ such that $Y' \cup Z'$ is a model of φ. This is addressed in the following lemma.

Lemma 2. *For every $Y' \subseteq Y$, $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1, \dots, n, \varphi\} \in \mathit{cl\text{-}naive}(CF)$ iff there is $Z' \subseteq Z$ such that $M = Y' \cup Z'$ is a model of φ.*

Proof. Let $S = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1, \dots, n, \varphi\}$.

First assume $S \in \mathit{cl\text{-}naive}(CF)$. Consider a cf_c-realization E of S. We have $\varphi \in E$ because φ is the unique argument having claim φ. Consequently, $a_i \notin E$ and thus each claim i is represented by x_i for some $x \in X \cup \bar{X}$. Let $Z' = \{z \in Z \mid z_i \in E\}$. Then $M = Y' \cup Z'$ is a model of φ: Consider an arbitrary clause cl_i. Since $\{1, \dots, n\} \subseteq S$, there is some argument with claim i in E, that is, either $a_i \in E$ or $x_i \in E$ or $\bar{x}_i \in E$ for some $x \in X$ (observe that $y_i \in E$ iff $y \in E$ and $\bar{y}_i \in E$ iff $\bar{y} \in E$, thus a further case distinction for $y \in Y$, $\bar{y} \in \bar{Y}$ is not required). We have that $a_i \notin E$ since $n \in S$ and for each argument b with $\mathit{claim}(b) = n$ we have $(a_i, b) \in R$. Thus there is $x \in X$ such that either $x_i \in E$ or $\bar{x}_i \in E$. In the former case, we have $x \in M$ and thus M satisfies cl_i, in the latter case $x \notin M$ and thus cl_i is satisfied. We obtain that M is a model of φ.

Now assume there is $Z' \subseteq Z$ such that $M = Y' \cup Z'$ is a model of φ. Let $E = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{x_i \mid x \in M\} \cup \{\bar{x}_i \mid x \notin M\} \cup \{\varphi\}$. E is conflict-free since $a_i \notin E$ for all $i < n$; other conflicts appear only between arguments x_i, $\bar{x}_j$ referring to the same atom x. Moreover, as M is a model of φ, we have that for each clause cl_i, there is either a positive literal $x \in cl_i$ with $x \in M$ or a negative literal $\bar{x} \in cl_i$ with $x \notin M$. Thus $\{1, \dots, n\} \subseteq \mathit{claim}(E)$; moreover, $Y' \cup \{\bar{y} \mid y \notin Y'\} \subseteq \mathit{claim}(E)$ and therefore $\mathit{claim}(E) = S$. S is a maximal cl-conflict-free claim-set since $S \cup \{c\} \notin \mathit{cf}_c(CF)$ for any $c \in (Y \cup \bar{Y}) \setminus S$ as each realization of $S \cup \{c\}$ contains y, $\bar{y}$ for some $y \in Y$. Thus $S \in \mathit{cl\text{-}naive}(CF)$.

□

We are now ready to prove the correctness of the reduction.

Lemma 3. *Ψ is valid iff the claim n is skeptically accepted with respect to cl-naive semantics in CF.*

Proof. Assume Ψ is not valid. Then there is $Y' \subseteq Y$ such that for all $Z' \subseteq Z$, $M = Y' \cup Z'$ does not satisfy φ. Let $S = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1, \dots, n\}$. Observe that S is i-conflict-free, witnessed by the cf_c-realization $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{a_1, \dots, a_n\}$. S is cl-naive since $S \cup \{\varphi\} \notin \mathit{cf}_c(CF)$ by (1) and $S \cup \{c\} \notin \mathit{cf}_c(CF)$ for any $c \in (Y \cup \bar{Y}) \setminus S$ as each realization of $S \cup \{c\}$ contains y, $\bar{y}$ for some $y \in Y$. Thus φ is not skeptically accepted with respect to cl-naive semantics in CF.

Assume φ is not skeptically accepted with respect to cl-naive semantics in CF. Then there is a set $S \in \mathit{cl\text{-}naive}(CF)$ such that $\varphi \notin S$. Observe that S contains $Y' \cup \{\bar{y} \mid y \notin Y\}$ for some $Y' \subseteq Y$ by construction. Let $Y' = S \cup Y$. We show that for all $Z' \subseteq Z$, $Y' \cup Z'$ is not a model of φ: Towards a contradiction assume there is $Z' \subseteq Z$ such that $M = Y' \cup Z'$ is a model of φ. By (1), $T = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{1, \dots, n, \varphi\} \in \mathit{cl\text{-}naive}(CF)$. Thus $T \supset S$ since $\varphi \notin S$, contradiction to S being cl-naive in CF. It follows that Ψ is not valid.

□

By the above lemma and the fact that the reduction can be performed in polynomial time we obtain Π_2^{P}-hardness.

Proposition 6. *$\mathit{Skept}^{CAF}_{\mathit{cl\text{-}naive}}$ is Π_2^{P}-hard.*

Hardness results for verification problems admit a higher complexity compared to AFs for all of the considered semantics. DP-hardness with respect to cl-preferred and cl-naive semantics will be shown by reductions from SAT-UNSAT; Σ_2^{P}- hardness with respect to i-semi-stable and i-stage semantics are by reductions from credulous reasoning for AFs with the respective semantics; the remaining hardness results are shown via reductions from appropriate decision problems for inherited semantics.

We first recall the standard reduction that provides the basis for DP-hardness of verification with respect to cl-preferred semantics and reappears in Section 5.

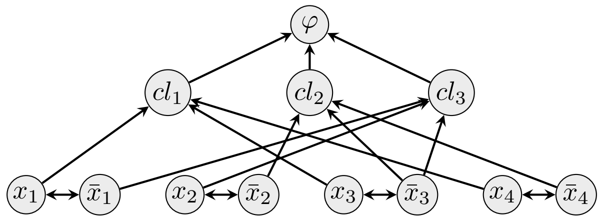


Figure 3: Reduction 2 for a formula which is given by the clauses $\{\{x_1, x_3, x_4\}, \{\bar{x}_3, \bar{x}_4, \bar{x}_2)\}, \{\bar{x}_1, \bar{x}_3, x_2\}\}$.

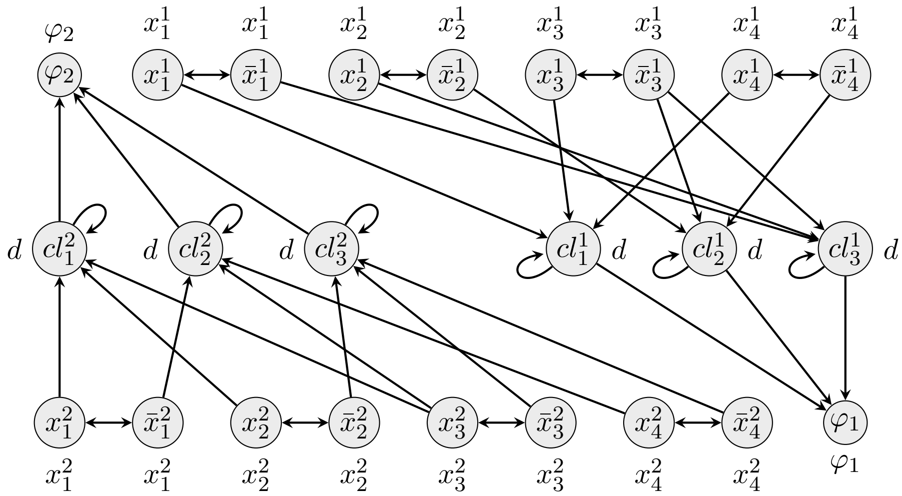


Figure 4: Reduction 3 for formulas (φ_1, φ_2) given by the sets of clauses $\{\{x_1^1, x_3^1, x_4^1\}, \{\bar{x}_3^1, \bar{x}_4^1, \bar{x}_2^1)\}, \{\bar{x}_1^1, \bar{x}_3^1, x_2^1\}\}$ and $\{\{x_1^2, x_2^2, x_3^2\}, \{\bar{x}_1^2, x_3^2, x_4^2\}, \{\bar{x}_2^2, \bar{x}_3^2, \bar{x}_4^2\}\}$

Reduction 2. *Let φ be given by a set of clauses $C = \{cl_1, \ldots, cl_n\}$ over atoms in X and let $\bar{X} = \{\bar{x} \mid x \in X\}$. We construct (A, R) with*

$$\begin{aligned} A &= X \cup \bar{X} \cup C \cup \{\varphi\} \\ R &= \{(x, cl) \mid cl \in C, x \in cl\} \cup \{(\bar{x}, cl) \mid cl \in C, \neg x \in cl\} \cup \\ &\quad \{(x, \bar{x}), (\bar{x}, x) \mid x \in X\} \cup \{(cl_i, \varphi) \mid i \leq n\} \end{aligned}$$

Intuitively, each conflict-free set of literal-arguments that defend the argument φ corresponds to a satisfying assignment of φ. An example of the reduction is given in Figure 3.

We next present a reduction from SAT-UNSAT to $Ver^{CAF}_{cl\text{-}prf}$ which shows DP-hardness. For a SAT-UNSAT instance (φ_1, φ_2) we apply Reduction 2 to both formulas and consider the disjoint union of the two resulting AFs.

Reduction 3. *Let (φ_1, φ_2) be an instance of SAT-UNSAT, where each of the propositional formulas φ_i (for $i = 1, 2$) is given over a set of clauses $C_i = \{cl_1^i, \ldots, cl_n^i\}$ over atoms in X_i. Moreover, we assume $X_1 \cap X_2 = \emptyset$. Let (A_i, R_i) be the AFs that we obtain when applying Reduction 2 to the formulas φ_i and adding attacks $\{(cl, cl) \mid cl \in C_i\}$. We construct the CAF $CF_{(\varphi_1,\varphi_2)} = (A_1 \cup A_2, R_1 \cup R_2, claim)$ with $claim(x) = claim(\bar{x}) = x$ for all $x \in X_i$, $claim(cl) = d$ for all $cl \in C_i$ and $claim(\varphi_i) = \varphi_i$. See Figure 4 for an illustrative example.*

We now observe that a formula φ_i is satisfiable iff $X_i \cup \{\varphi_i\}$ is a cl-preferred claim-set of $(A_i, R_i, claim)$ which yields the correctness of the reduction.

Lemma 4. *(φ_1, φ_2) is a valid SAT-UNSAT instance iff $X_1 \cup X_2 \cup \{\varphi_1\}$ is a cl-preferred claim-set of $CF_{(\varphi_1,\varphi_2)}$.*

Proof. We have to show that $X_1 \cup X_2 \cup \{\varphi_1\}$ is cl-preferred in $CF_{(\varphi_1,\varphi_2)}$ iff φ_1 is satisfiable and φ_2 is unsatisfiable. For the purpose of this proof we consider the CAF $CF_{(\varphi_1,\varphi_2)}$ as the disjoint union of the CAFs $CF_1 = (A_1, R_1, claim)$ and $CF_2 = (A_2, R_2, claim)$.

Since CF_1 and CF_2 are unconnected and have no common arguments (and thus $\mathit{cl\text{-}prf}(CF) = \{S \cup T \mid S \in \mathit{cl\text{-}prf}(CF_1), T \in \mathit{cl\text{-}prf}(CF_2)\}$), it suffices to show that

(a) φ_i is satisfiable iff $X_i \cup \{\varphi_i\}$ is a cl-preferred claim-set of CF_i, and

(b) φ_i is unsatisfiable iff X_i is a cl-preferred claim-set of CF_i.

We have that (b) follows from (a) since X_i is i-admissible in CF_i independently of the satisfiability of φ_i (for an adm_c-realization, consider $X' \cup \{\bar{x} \mid x \notin X'\}$ for any $X' \subseteq X_i$) and no argument $cl \in C_i$ can appear in an admissible set. We show φ_i is satisfiable iff $X_i \cup \{\varphi_i\}$ is a cl-preferred claim-set of CF_i:

Assume φ_i is satisfiable and consider a model M of φ_i. Let $E = M \cup \{\bar{x} \mid x \notin M\}$. We show that $E' = E \cup \{\varphi_i\}$ is admissible in (A_i, R'_i): First observe that E is admissible since each $a \in X_i \cup \bar{X}_i$ defends itself. Since M satisfies φ_i, we have that for any clause $cl \in C_i$, there is either $x \in cl$ with $x \in M$ or $\bar{x} \in cl$ with $x \notin M$, thus E attacks each $cl \in C$. Consequently, E defends φ_i; we conclude that E' is admissible in (A_i, R'_i). Moreover, $claim(E')$ is a subset-maximal i-admissible claim-set since $claim(E') = A_i \setminus \{d\}$, that is, $claim(E')$ contains every claim $c \in claim(A_i)$ which is assigned to non-self-attacking arguments. Thus $claim(E') = X_i \cup \{\varphi_i\}$ is cl-preferred in CF_i.

Now assume $X_i \cup \{\varphi_i\}$ is cl-preferred in CF_i. Let E be a adm_c-realization of $X_i \cup \{\varphi_i\}$ and let $M = E \cap X_i$. Consider an arbitrary clause $cl \in C_i$. Since $\varphi_i \in E$ is defended by E we have that E attacks cl, thus there is either an argument $x \in E$ such that $(x, cl) \in R'_i$ or an argument $\bar{x} \in E$ with $(\bar{x}, cl) \in R'_i$. In the former case, we have $x \in M$ and thus M satisfies cl, in the latter case $x \notin M$ and thus cl is satisfied. We obtain that M is a model of φ_i. □

By the above lemma and the fact that the reduction can be performed in polynomial time we obtain DP-hardness.

Proposition 7. *$Ver^{CAF}_{cl\text{-}prf}$ is DP-hard.*

DP-hardness of verification with respect to cl-naive semantics can be shown via a reduction from SAT-UNSAT by combining ideas from the previous propositions. As in Proposition 7, one constructs two independent frameworks CF_1, CF_2 representing the formulas (3-CNFs) φ_1, φ_2 with sets of clauses $C_1 = \{cl_1, \ldots, cl_m\}$ respectively $C_2 = \{cl_{m+1}, \ldots, cl_n\}$. The construction is similar to the one in Proposition 6, i.e., one introduces an argument with claim i for each literal in a clause $cl_i \in C_j$, an argument φ_j representing the respective formula and adds $|C_j|$ arguments with claims $1, \ldots, m$ respectively $m+1, \ldots, n$. One can show that $\{1, \ldots, n, \varphi_1\}$ is cl-naive in $CF_1 \cup CF_2$ iff φ_1 is satisfiable and φ_2 is unsatisfiable.

Reduction 4. *Let (φ_1, φ_2) be an instance of SAT-UNSAT, where each of the propositional formulas φ_j (for $j = 1, 2$) is given over a set of clauses C_j over atoms in X_j. Moreover, we assume $X_1 \cap X_2 = \emptyset$, $C_1 = \{cl_1, \ldots, cl_m\}$, $C_2 = \{cl_{m+1}, \ldots, cl_n\}$, and define $A'_1 = \{a_1, \ldots, a_m\}$ and $A'_2 = \{a_{m+1}, \ldots, a_n\}$.*

We construct the CAF $CF_{(\varphi_1,\varphi_2)} = (A, R, claim)$ with

$$\begin{aligned} A &= \{x_i \mid x \in cl_i, 1 \leq i \leq n\} \cup \{\bar{x}_i \mid \bar{x} \in cl_i, 1 \leq i \leq n\} \cup A'_1 \cup A'_2 \cup \{\varphi_1, \varphi_2\} \\ R &= \{(x_i, \bar{x}_j)(\bar{x}_j, x_i), \mid i, j \leq n\} \cup \{(a_i, \varphi_1), (\varphi_1, a_i) \mid i \leq m\} \cup \\ &\quad \{(a_i, \varphi_2), (\varphi_2, a_i) \mid m < i \leq n\} \end{aligned}$$

with $claim(x_i) = claim(\bar{x}_i) = claim(a_i) = i$ and $claim(\varphi_i) = \varphi_i$.

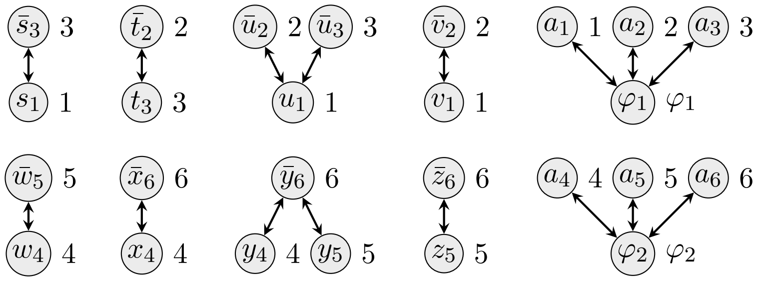


Figure 5: Reduction 4 for formulas (φ_1, φ_2) given by the sets of clauses $\{\{s, u, v\}, \{\bar{u}, \bar{v}, \bar{t})\}, \{\bar{s}, \bar{u}, t\}\}$ and $\{\{w, x, y\}, \{\bar{w}, y, z\}, \{\bar{x}, \bar{y}, \bar{z}\}\}$

Notice that the CAF $CF_{(\varphi_1,\varphi_2)}$ can be interpreted as the disjoint union of two CAFs, CF_1 represents φ_1 and CF_2 represents φ_2. See Figure 5 example illustrating the reduction.

Lemma 5. *(φ_1, φ_2) is a valid SAT-UNSAT instance iff $\{1, \dots, n, \varphi_1\} \in$ cl-naive(CF).*

Proof. For the purpose of this proof we consider the CAF $CF_{(\varphi_1,\varphi_2)}$ as disjoint union of two CAFs. To this end let CF_1 be the projection of $CF_{(\varphi_1,\varphi_2)}$ on the arguments $\{x_i \mid x \in cl_i, 1 \le i \le m\} \cup \{\bar{x}_i \mid \bar{x} \in cl_i, 1 \le i \le m\} \cup A'_1 \cup \{\varphi_1\}$ and CF_2 be the projection of $CF_{(\varphi_1,\varphi_2)}$ on the arguments
$\{x_i \mid x \in cl_i, m+1 \le i \le n\} \cup \{\bar{x}_i \mid \bar{x} \in cl_i, m+1 \le i \le n\} \cup A'_2 \cup \{\varphi_2\}$. Notice that $CF_{(\varphi_1,\varphi_2)} = CF_1 \cup CF_2$ and that CF_1 and CF_2 are isomorphic.

We show φ_1 is satisfiable and φ_2 is unsatisfiable iff
$\{1, \dots, n, \varphi_1\} \in$ *cl-naive*(CF) by proving

(a) φ_1 is satisfiable iff $\{1, \dots, m, \varphi_1\} \in$ *cl-naive*(CF_1).

(b) φ_2 is unsatisfiable iff $\{m+1, \dots, n\} \in$ *cl-naive*(CF_2).

Since CF_1, CF_2 are unconnected and $claim(A_1) \cap claim(A_2) = \emptyset$, we have $naive_c(CF) = \{S \cup T \mid S \in naive_c(CF_1), T \in naive_c(CF_2)\}$. Thus φ_1 is satisfiable and φ_2 is unsatisfiable iff $\{1, \dots, n, \varphi_1\} \in$ *cl-naive*(CF).

Proof of (a): First assume φ_1 is satisfiable and consider a model M of φ_1. Let $E = \{x_i \mid x \in M, i \le m\} \cup \{\bar{x}_i \mid x \notin M, i \le m\} \cup \{\varphi_1\}$. E is conflict-free by construction; moreover, $\varphi_1 \in claim(E)$ and $i \in claim(E)$ for each $i \le m$: For each clause $cl_i \in C_1$, there is either $x \in M \cap cl_i$ or $\bar{x} \in cl_i$ such that $x \notin M$, consequently there is either $x_i \in E$ with $claim(x_i) = i$ or $\bar{x}_i \in E$ with $claim(\bar{x}_i) = i$. We have shown that $\{1, \dots, m, \varphi_1\}$ has a conflict-free realization in CF_1.

Now assume $\{1, \dots, m, \varphi_1\} \in$ *cl-naive*(CF). Let E be a cf_c-realization of $\{1, \dots, m, \varphi_1\}$ and let $M = \{x \mid \exists i \le m : x_i \in E\}$. Now, consider an arbitrary clause $cl_i \in C_1$. Then E contains an argument with claim i, that is, either $x_i \in E$ or $\bar{x}_i \in E$. In the former case, $x \in M$ and thus cl_i is satisfied. In the latter case, $x \notin M$ as $\bar{x}_i$ is in conflict with all arguments x_j and thus cl_i is satisfied. We obtain that M is a model of φ_1 and thus φ_1 is satisfiable.

Proof of (b): First notice that $claim(A'_2) = \{m+1, \dots, n\}$ is i-conflict-free by construction. By (a), φ_2 is unsatisfiable iff $\{m+1, \dots, n, \varphi_2\} \notin$ *cl-naive*(CF'_2). We thus obtain φ_2 is unsatisfiable iff $\{m+1, \dots, n, \varphi_2\} \notin$ *cl-naive*(CF_2) iff $\{m+1, \dots, n\} \in$ *cl-naive*(CF_2). □

By the above lemma and the fact that the reduction can be performed in polynomial time we obtain DP-hardness.

Proposition 8. *$Ver^{CAF}_{cl\text{-}naive}$ is* DP-*hard.*

In the following, we show Σ_2^{P}-hardness of the verification problem for CAFs with respect to i-semi-stable and i-stage semantics, utilizing a reduction from the respective credulous acceptance problem for AFs.

Proposition 9. *$Ver^{CAF}_{sem_c}$ and $Ver^{CAF}_{stg_c}$ are Σ_2^{P}-hard.*

Proof. We present a proof for $Ver^{CAF}_{sem_c}$, the proof for $Ver^{CAF}_{stg_c}$ is analogous. For an instance (A, R), $b \in A$ of $Cred^{AF}_{sem}$, we construct a CAF $CF = (A', R, claim)$ with $A' = A \cup \{x\}$, $x \notin A$ and $claim(b) = c_1$, $claim(a) = c_2$ for all $a \in A' \setminus \{b\}$. Then, as the argument x is not involved in any attack, it is contained in every semi-stable extension of (A', R) and thus, as $claim(x) = c_2$, c_2 is contained in every i-semi-stable claim-set of CF. Furthermore, as CF contains only two claims, the only candidates for i-semi-stable claim-sets are $\{c_1, c_2\}$ and $\{c_2\}$. Moreover, as b is the only argument with claim c_1, $\{c_1, c_2\}$ is i-semi-stable iff b is contained in some semi-stable set of arguments in (A', R). Thus, b is credulously accepted in (A, R) w.r.t. semi-stable semantics iff $\{c_1, c_2\}$ is i-semi-stable in CF. Σ_2^{P}-hardness of $Ver^{CAF}_{sem_c}$ thus follows from known results for AFs. □

Finally, we provide hardness results for cl-semi-stable, τ-cl-stable and cl-stage semantics. We will present reductions from the verification problem of suitable inherited semantics. To that end, we consider the following translations.

Reduction 5. *For a CAF $CF = (A, R, claim)$, we define three translations:*

- $Tr_1(CF) = (A', R', claim')$ *with*

$$\begin{aligned} A' &= A \cup \{a' \mid a \in A\} \\ R' &= R \cup \{(a, a'), (a', a') \mid a \in A\} \end{aligned}$$

  and $claim'(a) = claim(a)$ for $a \in A$, $claim(a') = c_a$ for $a' \in \{a' \mid a \in A\}$ with fresh claims $c_a \notin claim(A)$.

- $Tr_2(CF) = (A', R'_2, claim')$ *with*

$$\begin{aligned} A' &= A \cup \{a' \mid a \in A\} \\ R'_2 &= R' \cup \{(a, b') \mid (a, b) \in R\}; \end{aligned}$$

  and $claim'$ as before.

- $Tr_3(CF) = (A', R'_3, claim')$ *with*

$$\begin{aligned} A' &= A \cup \{a' \mid a \in A\} \\ R'_3 &= R'_2 \cup \{(b, a) \mid (a, b) \in R\} \cup \{(a, b) \mid a \in A, (b, b) \in R\}; \end{aligned}$$

  and $claim'$ as before.

See Figure 6 for an example illustrating the translations. The following lemma states that (a) Tr_1 maps i-preferred semantics to cl-semi-stable semantics, (b) Tr_2 maps inherited to claim-level stable semantics, and (c) Tr_3 maps inherited to claim-level stage semantics. The proof can be found in the appendix.

Lemma 6. *For a CAF $CF = (A, R, claim)$,*

$$\begin{aligned} &prf_c(CF) = prf_c(Tr_1(CF)) = cl\text{-}sem(Tr_1(CF)), \\ &stb_c(CF) = stb_c(Tr_2(CF)) = cl\text{-}stb_\tau(Tr_2(CF)) \text{ for } \tau \in \{adm, cf\}, \\ &stg_c(CF) = stg_c(Tr_3(CF)) = cl\text{-}stg(Tr_3(CF)). \end{aligned}$$

Table 3: Complexity of inherited semantics for CAFs, full picture (results for i-semi-stable and i-stage semantics are new). Results that deviate from the corresponding AF results are highlighted in bold-face.

σ	$Cred_\sigma^{CAF}$	$Skept_\sigma^{CAF}$	Ver_σ^{CAF}	NE_σ^{CAF}
cf_c	in P	trivial	**NP**-c	in P
adm_c	NP-c	trivial	**NP**-c	NP-c
stb_c	NP-c	coNP-c	**NP**-c	NP-c
$naive_c$	in P	**coNP**-c	**NP**-c	in P
prf_c	NP-c	Π_2^P-c	$\boldsymbol{\Sigma_2^P}$-c	NP-c
sem_c	Σ_2^P-c	Π_2^P-c	$\boldsymbol{\Sigma_2^P}$-c	NP-c
stg_c	Σ_2^P-c	Π_2^P-c	$\boldsymbol{\Sigma_2^P}$-c	in P

Table 4: Complexity of claim-based semantics for CAFs. Results that deviate from the corresponding AF results are highlighted in bold-face; results that deviate from those w.r.t. inherited semantics are underlined.

σ	$Cred_\sigma^{CAF}$	$Skept_\sigma^{CAF}$	Ver_σ^{CAF}	NE_σ^{CAF}
$cl\text{-}stb_{adm}$	NP-c	coNP-c	**NP**-c	NP-c
$cl\text{-}stb_{cf}$	NP-c	coNP-c	**NP**-c	NP-c
$cl\text{-}naive$	in P	$\underline{\boldsymbol{\Pi_2^P}\text{-c}}$	$\underline{\textbf{DP}\text{-c}}$	in P
$cl\text{-}prf$	NP-c	Π_2^P-c	$\underline{\textbf{DP}\text{-c}}$	NP-c
$cl\text{-}sem$	Σ_2^P-c	Π_2^P-c	$\boldsymbol{\Sigma_2^P}$-c	NP-c
$cl\text{-}stg$	Σ_2^P-c	Π_2^P-c	$\boldsymbol{\Sigma_2^P}$-c	in P

Lower bounds for Ver_σ^{CAF}, $\sigma \in \{cl\text{-}stb_{adm}, cl\text{-}stb_{cf}, cl\text{-}sem, cl\text{-}stg\}$, thus follow from the results of the respective inherited semantics: For a given CAF $CF = (A, R, claim)$ and a set of claims $S \subseteq claim(A)$, one can check $S \in \sigma'_c(CF)$, $\sigma' \in \{stb, prf, stg\}$, by applying the respective translation and checking whether S is a σ-realization in the resulting CAF.

Proposition 10. *Ver_σ^{CAF} is NP-hard for $\sigma \in \{cl\text{-}stb_{adm}, cl\text{-}stb_{cf}\}$ and Σ_2^P-hard for $\sigma \in \{cl\text{-}sem, cl\text{-}stg\}$.*

Proof. The NP-hardness of Ver_σ^{CAF} for $\sigma \in \{cl\text{-}stb_{adm}, cl\text{-}stb_{cf}\}$ is by the fact that Ver_{stb_c} is NP-hard and translation Tr_2. The Σ_2^P-hardness of $Ver_{cl\text{-}sem}^{CAF}$ is by the fact fact that Ver_{prf_c} is Σ_2^P-hard and translation Tr_1. Finally, the Σ_2^P-hardness of $Ver_{cl\text{-}stg}^{CAF}$ is by the fact fact that Ver_{stg_c} is Σ_2^P-hard and translation Tr_3. □

This concludes our complexity analysis of general CAFs. The full complexity landscape is summarized in Tables 3 & 4. Table 3 shows the results for inherited semantics (together with the results of [12]) while Table 4 shows the results for claim-based semantics.

4.2 Complexity Results for well-formed CAFs

We now turn to the complexity of well-formed CAFs. First observe that all upper bounds from the previous section carry over since well-formed CAFs are a special case of CAFs. It remains to give improved upper bounds for verification with respect to all of the considered semantics as well as for $Skept_{cl\text{-}naive}^{wf}$. The latter also requires a genuine hardness proof as it remains harder than the corresponding problem for AFs even in the well-formed case. For the remaining semantics, we obtain hardness results from the corresponding problems for AFs since they constitute a special case of the respective problems for CAFs.

We first discuss improved upper bounds for verification. For preferred as well as for both variants of cl-stable semantics, membership is immediate by the corresponding results for inherited semantics as the respective semantics collapse in the well-formed case [8].

Proposition 11. *Ver^{wf}_{σ} is in P for $\sigma \in \{\textit{cl-stb}_{cf}, \textit{cl-stb}_{adm}\}$ and coNP-complete for $\sigma = \textit{cl-prf}$.*

For the remaining semantics, we exploit the following observation [12].

Lemma 7. *Let $CF = (A, R, claim)$ be well-formed. For $S \subseteq claim(A)$, let*

$$E_0(S) = \{a \in A \mid cl(a) \in S\}$$
$$E_1(S) = E_0(S) \setminus E_0(S)^+_{(A,R)}$$
$$E_2(S) = \{a \in E_1(S) \mid b \in E_1(S)^+_{(A,R)} \text{ for all } (b, a) \in R\}.$$

Then $S \in cf_c(CF)$ iff $S = claim(E_1(S))$ and $S \in adm_c(CF)$ iff $S = claim(E_2(S))$.

To check whether a set $S \subseteq claim(A)$ is cl-naive in a given well-formed CAF $CF = (A, R, claim)$, we utilize Lemma 7 to test (i) $S \in cf_c(CF)$ and (ii) $S \cup \{c\} \notin cf_c(CF)$ for all $c \in claim(A) \setminus S$, which yields a poly-time procedure for Ver^{wf}_{naive}.

Proposition 12. *Ver^{wf}_{naive} is in P.*

For inherited as well as claim-level semi-stable and stage semantics, we first compute $E_1(S)$, respectively $E_2(S)$ in P (cf. Lemma 7). For cl-semi-stable (cl-stage) semantics, utilize Lemma 1 to check in coNP whether $E_2(S)$ ($E_1(S)$) realizes a cl-semi-stable (cl-stage) claim set; similarly, for i-semi-stable (i-stage) semantics, we check that $E_2(S) \in sem((A, R))$ ($E_1(S) \in stg((A, R))$), which is known to be coNP-complete.

Proposition 13. *Ver^{wf}_{σ} is in coNP for $\sigma \in \{\textit{cl-sem}, \textit{cl-stg}, sem_c, stg_c\}$.*

Finally, we will discuss coNP-completeness of skeptical reasoning in well-formed CAFs w.r.t. cl-naive semantics. To show hardness, we make use of a small adaption of the standard reduction (cf. Reduction 2) by removing the argument φ and all associated attacks.

Proposition 14. *$Skept^{wf}_{\textit{cl-naive}}$ is coNP-complete.*

Proof. For a well-formed CAF $CF = (A, R, claim)$, one can verify skeptical acceptance of a claim $c \in claim(A)$ by (1) guessing a set $E \subseteq A$ such that $c \notin claim(E)$; (2) checking if $claim(E)$ is a cl-naive claim-set of CF. The latter can be verified in polynomial time, yielding a NP-procedure for the complementary problem.

Hardness can be shown via a reduction from UNSAT: For a formula φ with clauses $C = \{cl_1, \ldots, cl_n\}$ over the atoms X, let (A', R') be as in Reduction 2. We define $CF = (A, R, claim)$ with $A = A' \setminus \{\varphi\}$ and $R = R' \setminus \{(cl_i, \varphi) \mid i \leq n\}$, moreover, we set $claim(x) = x$, $claim(\bar{x}) = \bar{x}$, and $claim(cl_i) = \bar{\varphi}$. See Figure 7 for an illustrative example of the reduction. Observe that CF is well-formed. We show φ is satisfiable iff $\bar{\varphi}$ is not skeptically accepted in CF.

In case φ is satisfiable, then there is a model $M \subseteq X$ of φ. Consider $E = M \cup \{\bar{x} \mid x \notin M\}$, which is conflict-free and cannot be extended by any argument cl_i assigned with claim $\bar{\varphi}$: Indeed, since each clause cl_i is satisfied by M, there is either a positive literal $x \in cl_i$ with $x \in M$ or a negative literal $\bar{x} \in cl_i$ with $x \notin M$, thus cl_i is attacked by E in (A, R). Moreover, we have that for each $x \in X$, either $x \in E$ (and thus $x \in claim(E)$) or $\bar{x} \in E$ (and thus $\bar{x} \in claim(E)$) and $(x, \bar{x}) \in R$. Consequently, $claim(E)$ is maximal among i-conflict-free

Table 5: Complexity of inherited semantics in well-formed CAFs, full picture (results for i-semi-stable and i-stage semantics are new). Results that deviate from general CAFs (cf. Table 3) are highlighted in bold-face.

σ	$Cred^{wf}_\sigma$	$Skept^{wf}_\sigma$	Ver^{wf}_σ	NE^{wf}_σ
cf_c	in P	trivial	in P	in P
adm_c	NP-c	trivial	in P	NP-c
stb_c	NP-c	coNP-c	in P	NP-c
$naive_c$	in P	**coNP**-c	in P	in P
prf_c	NP-c	Π_2^P-c	coNP-c	NP-c
sem_c	Σ_2^P-c	Π_2^P-c	**coNP**-c	NP-c
stg_c	Σ_2^P-c	Π_2^P-c	**coNP**-c	in P

Table 6: Complexity of claim-based semantics in well-formed CAFs. Results that deviate from general CAFs (cf. Table 4) are highlighted in bold-face.

σ	$Cred^{wf}_\sigma$	$Skept^{wf}_\sigma$	Ver^{wf}_σ	NE^{wf}_σ
$cl\text{-}stb_{cf}$	NP-c	coNP-c	in **P**	NP-c
$cl\text{-}stb_{adm}$	NP-c	coNP-c	in **P**	NP-c
$cl\text{-}naive$	in P	**coNP**-c	in **P**	in P
$cl\text{-}prf$	NP-c	Π_2^P-c	**coNP**-c	NP-c
$cl\text{-}sem$	Σ_2^P-c	Π_2^P-c	**coNP**-c	NP-c
$cl\text{-}stg$	Σ_2^P-c	Π_2^P-c	**coNP**-c	in P

claim-sets and thus $claim(E) \in cl\text{-}naive(CF)$. It follows that $\bar{\varphi}$ is not skeptically accepted in CF.

Now assume $\bar{\varphi}$ is not skeptically accepted in CF, then there is a set $S \in cl\text{-}naive(CF)$ such that $\bar{\varphi} \notin S$. For a cf_c-realization E of S, we have $M = E \cap X$ is a model of φ: Consider an arbitrary clause cl_i. As $\bar{\varphi} \notin S$ we have that E attacks cl_i, thus there is either an argument $x \in E$ such that $(x, cl_i) \in R$ or an argument $\bar{x} \in E$ with $(\bar{x}, cl_i) \in R$. In the former case, we have $x \in M$ and thus M satisfies cl_i, in the latter case $\bar{x} \notin M$ and thus cl_i is satisfied. We obtain that M is a model of φ. □

This concludes our complexity analysis of well-formed CAFs. All the results are summarized in Tables 5 & 6.

5 Complexity of Concurrence

This section examines the complexity of deciding concurrence of the different variants of the considered semantics and studies a claim-based variant of the coherence problem.

The inherent difference of maximization on argument- respectively claim-level in CAFs has been already discussed by [8] who showed that also for well-formed CAFs, claim-level and inherited versions of semi-stable and stage semantics potentially yield different claim-sets. In this section, we first consider the complexity of Con^{CAF}_σ and Con^{wf}_σ, that is: Given a (well-formed) CAF CF and a semantics σ, how hard is it to decide whether $\sigma_c(CF) = cl\text{-}\sigma(CF)$? Our results are summarized in Table 7 and show that deciding concurrence is in general computationally hard; observe that for semi-stable and stage semantics, the problem is complete for the third level of the polynomial hierarchy. For preferred and stable semantics

Table 7: Complexity of deciding Con_σ^{CAF} and Con_σ^{wf}.

	prf	*naive*	stb_τ	*sem*	*stg*
Con_σ^{CAF}	Π_2^P-c	coNP-c	Π_2^P-c	Π_3^P-c	Π_3^P-c
Con_σ^{wf}	trivial	coNP-c	trivial	Π_2^P-c	Π_2^P-c

on the other hand, the question becomes trivial for well-formed CAFs as the claim-based versions of this semantics coincide with their inherited counter-parts.

We furthermore show that deciding whether $cl\text{-}stb_{cf}(CF) = cl\text{-}stb_{adm}(CF)$ for a given CAF CF is Π_2^P-complete and conclude the section with a brief discussion of the well-known coherence problem when applied to claim-based semantics. However, let us start with the collection of results concerning concurrence which will be proven in the forthcoming two subsections.

Theorem 1. *The complexity results depicted in Table 7 hold.*

5.1 Concurrence of General CAFs

We start with a rather straight-forward observation for preferred and naive semantics which will be useful for both membership and hardness arguments. The distinguishing factor of inherited and claim-level variants of preferred and naive semantics is *incomparability*: a set of sets $\mathcal{X} = \{X_1, \dots, X_n\}$ is incomparable iff $X_i \not\subseteq X_j$ for all $i, j \leq n$. Claim-level variants of both semantics return incomparable sets of claim-extensions since maximization is performed on claim-level. We show next that the two different variants of preferred and naive semantics coincide iff the inherited variants return incomparable sets as well.

Proposition 15. *For a CAF $CF = (A, R, claim)$, for $\sigma \in \{prf, naive\}$, $\sigma_c(CF) = cl\text{-}\sigma(CF)$ if and only if $\sigma_c(CF)$ is incomparable.*

Proof. Let $\sigma = prf$ (the proof for $\sigma = naive$ is analogous). Assume $prf_c(CF)$ is incomparable and let $S \in prf_c(CF)$. Then $S \in adm_c(CF)$. Now assume there is $T \in adm_c(CF)$ with $T \supset S$. Consider a adm_c-realization E of T in CF and let $E' \in prf((A, R))$ with $E \subseteq E'$. But then $claim(E') \in prf_c(CF)$ and $claim(E') \supseteq T \supset S$, contradiction to $prf_c(CF)$ being incomparable. ☐

To get upper bounds for preferred and naive semantics, it thus suffices to verify incomparability of $\sigma_c(CF)$. We give a Σ_2^P (NP resp.) procedure for the complementary problem: Guess $E, G \subseteq A$ and check (i) $E, G \in \sigma((A, R))$ and (ii) $claim(E) \subset claim(G)$. The former is in coNP for *prf* (in P for *naive*) by Table 1.

Membership for the remaining semantics is by the following generic guess and check procedure for the complementary problem: To show for a given CAF $CF = (A, R, claim)$ that $\sigma_c(CF) \neq cl\text{-}\sigma(CF)$ one first guesses a set of claims $S \subseteq claim(A)$ and checks whether $S \in \sigma_c(CF)$ and $S \notin cl\text{-}\sigma(CF)$ or vice versa. The complexity of the procedure thus follows from the corresponding results for verification with respect to the considered semantics, i.e. NP-membership for the stable semantics; Σ_2^P-membership for semi-stable and stage semantics, cf. Tables 3 and 4.

Before turning to the results for the matching lower bounds in general CAFs, let us point out that for all except naive semantics, deciding concurrence admits a lower complexity for well-formed CAFs than for general CAFs. In the preliminary version of this paper, we have proven coNP-hardness of deciding concurrence for general CAFs while the complexity of this problem for well-formed CAFs has been left open. This gap has been closed recently [25] by showing that coNP-hardness holds even in the well-formed case. Due to this novel insights, we omit the original hardness proof for general CAFs presented in [16] and refer the interested reader to [25].

Proposition 16. *Con_{prf}^{CAF} is Π_2^{P}-hard.*

Proof. We present a reduction from $Skept_{prf}^{AF}$: Given an instance (A, R), $a \in A$ from $Skept_{prf}^{AF}$. W.l.o.g. we can assume that the preferred extensions of (A, R) are non-empty (otherwise add an isolated argument). We construct $CF = (A', R', claim)$ with $A' = A \cup \{i, j\}$, $R' = R \cup \{(j, b), (b, j) \mid b \in A\}$, and $claim(a) = claim(j) = c_1$, $claim(b) = c_2$ for $b \in (A \setminus \{a\}) \cup \{i\}$. Then $prf((A', R')) = \{E \cup \{i\} \mid E \in prf((A, R))\} \cup \{\{i, j\}\}$ since the argument i is isolated and thus appears in each extension; moreover, j mutually attacks each argument $b \in A$. For all extensions $D \in prf((A', R'))$ with $a \in D$ we have $claim(D) = \{c_1, c_2\}$; for all extensions $D \in prf((A', R'))$, $D \neq \{i, j\}$, with $a \notin D$, we have $claim(D) = \{c_2\}$; moreover, $claim(\{i, j\}) = \{c_1, c_2\}$ and thus we have $\{c_1, c_2\} \in prf_c(CF)$ independently of the considered instance. Thus a is not skeptically accepted in (A, R) with respect to preferred semantics iff $\{c_2\} \in prf_c(CF)$ iff $prf_c(CF)$ is not incomparable. Applying Proposition 15 concludes the proof. □

Next we present our Π_2^{P}-hardness proof for claim-level stable semantics. We will make use of the following reduction.

Reduction 6. *Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT_2^{\forall}$, where φ is given by a set of clauses $C = \{cl_1, \ldots, cl_n\}$ over atoms in $X = Y \cup Z$ and let (A, R) be as in Reduction 2. We define a CAF $(A', R', claim)$ with*

$$A' = A \setminus \{\varphi\}$$
$$R' = (R \cup \{(cl_i, cl_i) \mid i \leq n\}) \setminus \{(cl_i, \varphi) \mid i \leq n\}$$

and $claim(y) = y$, $claim(\bar{y}) = \bar{y}$, $claim(v) = claim(cl_i) = c$ for $i \leq n$ and $v \in Z \cup \bar{Z}$.

See Figure 8 for an illustrative example of the reduction.

Proposition 17. *$Con_{stb_\tau}^{CAF}$, $\tau \in \{cf, adm\}$ is Π_2^{P}-hard.*

Proof. We present a reduction from $QSAT_2^{\forall}$. Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT_2^{\forall}$, where φ is given by a set of clauses $C = \{cl_1, \ldots, cl_n\}$ over atoms in $X = Y \cup Z$. Let (A, R) be as in Reduction 6.

We will first show that (a) $cl\text{-}stb_\tau = \{Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \mid Y' \subseteq Y\}$: Each τ-cl-stable claim-set S contains either y or $\bar{y}$ by construction; moreover, $c \in S$ since c is not defeated by any conflict-free set of arguments $E \subseteq A$, thus each τ-cl-stable claim-set is of the form $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\}$ for some $Y' \subseteq Y$. Moreover, each such set is stb_τ-realizable, since for any $Y' \subseteq Y$, $z \in Z$, the set $E = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{z\}$ is admissible (conflict-free) in (A, R') and attacks every $a \in A$ such that $claim(a) \notin claim(E)$.

We show Ψ is valid iff $stb_c(CF) = cl\text{-}stb_\tau(CF)$.

Assume Ψ is valid. Let $Y' \subseteq Y$. Then there is $Z' \subseteq Z$ such that φ is satisfied by $M = Y' \cup Z'$. Let $E = M \cup \{\bar{x} \mid x \notin M\}$. Since M satisfies each clause cl_i, there is either $x \in cl_i$ with $x \in M$ or there is $\bar{x} \in cl_i$ with $x \notin M$. It follows that each cl_i, $i \leq n$, is attacked by E. Since E is also conflict-free we have shown that E is a stable extension of (A, R) and therefore $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \in stb_c(CF)$. As Y' was arbitrary, we have that $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \in stb_c(CF)$ for all $Y' \subseteq Y$. We conclude that $stb_c(CF) = cl\text{-}stb_\tau(CF)$ by (a).

Assume $stb_c(CF) = cl\text{-}stb_\tau(CF)$. Let $Y' \subseteq Y$. By (a) we have that $S = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \in cl\text{-}stb_\tau(CF) = stb_c(CF)$. Consider a stb_c-realization E of S and let $Z' = E \cap Z$.

Then $M = Y' \cup Z'$ satisfies φ: Consider an arbitrary clause cl_i. As E attacks cl_i there is either an argument $x \in E$ with $(x, cl_i) \in R$ or an argument $\bar{x} \in E$ with $(\bar{x}, cl_i) \in R$. In the former case, $x \in cl_i$ and $x \in M$ and thus cl_i is satisfied; in the latter case, $\bar{x} \in cl_i$ and $x \notin M$ and thus cl_i is satisfied. Thus M is a model of φ. We have shown that for every $Y' \subseteq Y$, there is $Z' \subseteq Z$ such that $Y' \cup Z'$ satisfies φ. It follows that Ψ is valid. $\square$

We finally arrive at the Π_3^{P}-hardness proofs for concurrence in the case of semi-stable and stage semantics. We reduce from $QSAT_3^{\exists}$. Our formulae are of the form $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ for a CNF φ over variables in $X \cup Y \cup Z$. The basis for our reduction builds the standard reduction (cf. Reduction 2). We will deal with the arguments corresponding to the different groups of literals over X, Y, and Z as follows:

- For each argument $l \in \{x, \neg x\}$ corresponding to a literal over atoms in X, we introduce a self-attacking dummy argument d_l which is attacked by l. Moreover, each argument is assigned its own name; i.e., argument l has claim l.

  In this way, we ensure that we can treat different truth assignments for atoms in X separately in the CAF (the dummy arguments indicate whether x or $\neg x$ is contained in the extension because only one of them is contained in the range). Moreover, each truth assignment gives rise to a distinct claim-extension.

- For arguments corresponding to literals over Y, we proceed similarly and introduce dummy arguments. However, we do not distinguish between atoms and their negation. We do so by assigning the argument corresponding to atom y and the argument corresponding to its negation the same claim y, for each atom $y \in Y$.

  Again, we encode the truth assignments for atoms in Y with the dummy arguments. However, now we cannot distinguish the truth assignments when looking only at the claim-extensions of the CAF.

- Arguments associated to literals over Z do not distinguish between atoms and their negation. We assign each pair of arguments corresponding to an atom $z \in Z$ and its negation the same claim z.

  For atoms over Z, it does not matter whether we choose the argument corresponding to a given atom or its negation. As the arguments are existentially quantified it suffices to consider *some* satisfying assignment.

We furthermore extend the basic reduction with attacks on and from the argument corresponding to φ. First, we add an argument $\bar{\varphi}$ that symmetrically attacks φ. In this way, we ensure that φ appears in the (claim-)range of each extension. Second, we add two self-attacking arguments d_1 and d_2 with the same claim d. Here, only one of them (d_1) is attacked by φ. This gadget is crucial to separate claim-level and inherited semantics: On argument-level, it is always better to include φ instead of $\bar{\varphi}$ in the extension whenever possible since the argument-based range contains d_1 if φ is contained in the extension. The claim-range of an admissible (conflict-free) set, however, does not distinguish between an extension containing φ and an extension containing $\bar{\varphi}$ since not all occurrences of d are attacked.

Below, we state the formal definition.

Reduction 7. *Let $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ be an instance of $QSAT_3^{\exists}$, where φ is given by a set of clauses $\mathcal{C} = \{cl_1, \dots, cl_n\}$ over atoms in $V = X \cup Y \cup Z$. We can assume that there is a variable $y_0 \in Y$ with $y_0 \in cl_i$ for all $i \leq n$ (otherwise we can add such a y_0 without* *changing the validity of* Ψ*). Let* (A, R) *be the AF constructed from* φ *as in Reduction 2. We define* $CF = (A', R', claim)$ *with*

$$\begin{aligned} A' &= A \cup \{d_1, d_2, \bar{\varphi}\} \cup \{d_v, d_{\bar{v}} \mid v \in X \cup Y\} \\ R' &= R \cup \{(a, d_a), (d_a, d_a), \mid a \in X \cup \bar{X} \cup Y \cup \bar{Y}\} \cup \\ &\quad \{(\varphi, \bar{\varphi}), (\bar{\varphi}, \varphi), (\varphi, d_1)\} \cup \{(d_i, d_j) \mid i, j \leq 2\} \end{aligned}$$

and $claim(v) = claim(\bar{v}) = v$ *for* $v \in Y \cup Z$*;* $claim(cl_i) = \bar{\varphi}$ *for* $i \leq n$*;* $claim(d_i) = d$ *for* $i = 1, 2$*;* $claim(a) = a$ *otherwise.*

An illustrative example of the reduction is given in Figure 9.

The following lemma deals with the structure of the cl-semi-stable and i-semi-stable claim-sets of the constructed CAF CF.

Lemma 8. *Let* $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ *be an instance of* $QSAT_3^{\exists}$ *and let* $CF = (A, R, claim)$ *be as in Reduction 7. Then for all* $E \in sem((A, R))$*,*

1. $\varphi \in E \Leftrightarrow \bar{\varphi} \notin E$*;*
2. $\varphi \in E \Leftrightarrow E^{\oplus}_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_2\})$*;*
3. $\bar{\varphi} \in E \Leftrightarrow \mathcal{C} \cap E \neq \emptyset$*;*
4. $\bar{\varphi} \in E \Leftrightarrow E^{\oplus}_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_1, d_2\})$*.*

Proof. Let $F = (A, R)$ and first observe that (1) is immediate by construction.

For (2), first assume $\varphi \in E$. Then $\bar{\varphi}, d_1 \in E^{\oplus}_F$ since $\varphi \in E$; also, $\varphi \in E$ only if E defends φ against each cl_i, $i \leq n$, thus each cl_i is attacked by E; moreover, each $a \in V \cup \bar{V}$ is either contained or attacked by E, otherwise, $D = E \cup \{a\}$ is admissible in (A, R) with $D^{\oplus}_F \supset E^{\oplus}_F$, contradiction to $E \in sem((A, R))$. Thus $V \cup \bar{V} \in E^{\oplus}_F$ and $d_a \in E^{\oplus}_F$ for $a \in E \cap (X \cup \bar{X} \cup Y \cup \bar{Y})$. In case $E^{\oplus}_F = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_2\})$, we have $\varphi \in E$ since φ is the only argument attacking d_1.

To show (3), first assume $\bar{\varphi} \in E$. Towards a contradiction assume $\mathcal{C} \cap E = \emptyset$. Then $D = (E \cup \{\varphi\}) \setminus \{\bar{\varphi}\}$ is admissible in (A, R) and $D^{\oplus}_F$ is a proper subset of $E^{\oplus}_F$, contradiction to E being semi-stable in (A, R). It follows that $\mathcal{C} \cap E \neq \emptyset$. The other direction is immediate since $\mathcal{C} \cap E \neq \emptyset$ implies $\varphi \notin E$. By (1) we obtain $\bar{\varphi} \in E$.

To show (4) let us again assume $\bar{\varphi} \in E$. Then $\varphi \in E^{+}_F$; moreover, each $a \in V \cup \bar{V}$ is either contained in E or attacked by E, otherwise, $D = (E \cup \{a\}) \setminus \{cl_i \mid i \leq n, (a, cl_i) \in R\}$ is admissible in (A, R) and satisfies $D^{\oplus}_F \supset E^{\oplus}_F$, contradiction to $E \in sem((A, R))$. We thus have $V \cup \bar{V} \in E^{\oplus}_F$ and $d_a \in E^{\oplus}_F$ for $a \in E \cap (X \cup \bar{X} \cup Y \cup \bar{Y})$. Also, each cl_i is either attacked by E or defended by E (by (3), there is at least one $i \leq n$ such that $cl_i \in E$). The other direction follows since $d_1 \notin E^{\oplus}_F$ and thus $\varphi \notin E$. □

Next we provide some properties for the reduction making use of the observation that for any instance of $QSAT_3^{\exists}$, each i-semi-stable and each cl-semi-stable claim-set in the resulting CAF is of the form $S_{X'} \cup \{e\}$ where

$$S_{X'} = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z$$

for some $X' \subseteq X$ and for $e \in \{\varphi, \bar{\varphi}\}$; in fact, it can be shown that each such set is *cl-sem*-realizable. Note that this is not the case for i-semi-stable semantics (as a counter-example, consider $e = \bar{\varphi}$ and $X = \{x\}$ in Figure 9).

Lemma 9. *Let $CF = (A, R, claim)$ be as in Reduction 7 for an instance $\exists X \forall Y \exists Z \varphi(X, Y, Z)$ of $QSAT_3^\exists$. Then,*

$$\{S_{X'} \cup \{\varphi\} \mid X' \subseteq X\} \subseteq sem_c(CF) \subseteq cl\text{-}sem(CF) = \{S_{X'} \cup \{e\} \mid X' \subseteq X, e \in \{\varphi, \bar{\varphi}\}\}$$

Proof. Let $F = (A, R)$. To prove the statement we will first show that (i) each cl-semi-stable and each i-semi-stable claim-set is of the form $S_{X'} \cup \{e\}$ for some $X' \subseteq X$ and for $e \in \{\varphi, \bar{\varphi}\}$. As $sem_c(CF) \subseteq prf_c(CF)$ and $cl\text{-}sem(CF) \subseteq prf_c(CF)$, it suffices to prove the statement for each i-preferred claim-set S. First observe that S cannot contain both $a, \bar{a}$ for $a \in X \cup \{\varphi\}$ since there is no cf_c-realization containing both $a, \bar{a}$. As each other claim in $claim(A) \setminus (V \cup \bar{V} \cup \{\varphi, \bar{\varphi}\})$ is self-attacking, it remains to show that $S_{X'} \cup \{e\} \subseteq S$ for some $X' \subseteq X$, $e \in \{\varphi, \bar{\varphi}\}$: (a) S contains $X' \cup \{\bar{x} \mid x \notin X'\}$ for some for some $X' \subseteq X$: Assume there is $x \in X$ such that $x, \bar{x} \notin S$. Consider a prf_c-realization E of S and let $D = E \cup \{x\}$. D is conflict-free since $\bar{x}, d_x \notin E$, moreover, $cl_i \notin E$ for each clause cl_i with $(x, cl_i) \in R$, since cl_i is not defended against the attack (x, cl_i). Also, D is admissible since E does not contain the only attacker $\bar{x}$ of x and $D \supset E$, contradiction to E being preferred in (A, R). (b) S contains $Y \cup Z$: Assume there is $v \in Y \cup Z$ such that $v \notin S$. Consider a prf_c-realization E of S and let $D = E \cup \{v\}$. D is admissible since $\bar{v} \notin E$ by assumption $v \notin S$ and $D \supset E$, contradiction to E being preferred in (A, R). (c) S contains either φ or $\bar{\varphi}$: Towards a contradiction, assume $\varphi, \bar{\varphi} \notin S$. Consider a prf_c-realization E of S and let $D = E \cup \{\bar{\varphi}\}$. D is admissible since $\varphi \notin E$ and $D \supset E$, contradiction to E being preferred in (A, R). We thus have shown that each inherited as well as each claim-level semi-stable claim-set is of the form $S_{X'} \cup \{e\}$, $e \in \{\varphi, \bar{\varphi}\}$, for some set $X' \subseteq X$.

Next we show that each set of the form $S_{X'} \cup \{\varphi\}$ is i-semi-stable in CF. Fix some set $X' \subseteq X$ and let $E = X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\} \cup \{\varphi\}$ for some $Z' \subseteq Z$ and $Y' \subseteq Y$ with $y_0 \in Y'$. E defends φ as $y_0 \in cl_i$ for all $i \leq n$, thus E is admissible. Moreover, E is semi-stable since $E_F^\oplus = V \cup \bar{V} \cup \{d_a \mid a \in E \cap (X \cup \bar{X} \cup Y \cup \bar{Y})\} \cup \mathcal{C} \cup \{\varphi, \bar{\varphi}, d_1\}$ is subset-maximal: Assume there is $D \in adm((A, R))$ with $D_F^\oplus \supset E_F^\oplus$, that is, there is $e \in \{d_2\} \cup \{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\}$ such that $e \in D_F^\oplus$; in particular, $e \in D_F^+$ because all considered arguments are self-attacking. Observe that $d_2 \notin D_F^+$ since its only attacker is self-attacking. In case $e = d_a$ for some $a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E$ we have $a \in D$ and $\bar{a} \in D$ and thus D is conflicting, contradiction to D being conflict-free. Thus we have shown that $claim(E) = S_{X'} \cup \{\varphi\}$ is i-semi-stable.

It remains to prove that *each* set of the form $S_{X'} \cup \{e\}$ for some $X' \subseteq X$, $e \in \{\varphi, \bar{\varphi}\}$ is cl-semi-stable in CF. Let $X' \subseteq X$. We first show that $S_{X'} \cup \{\bar{\varphi}\}$ is cl-semi-stable in CF. Consider some $Y' \subseteq Y$, $Z' \subseteq Z$ and let $\mathcal{C}' \subseteq \mathcal{C}$ denote the set of clauses cl_i which are not attacked by $X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\}$. Let $E = X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\} \cup \mathcal{C}' \cup \{\bar{\varphi}\}$. Then E is admissible, $claim(E) = S_{X'} \cup \{\bar{\varphi}\}$, and $\nu_{CF}(E) = \{d_a \mid a \in X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\}\} \cup \{\varphi\}$. Thus $claim(E) \cup \nu_{CF}(E)$ is subset-maximal among admissible sets since it contains every claim $c \in claim(A)$ which is assigned to non-self-attacking arguments; moreover, it contains a maximal set of claims among $\{d_v \mid v \in V \cup \bar{V}\}$ since it contains precisely one of $d_v, d_{\bar{v}}$ for each $v \in V$; furthermore observe that $d \notin \nu_{CF}(E)$ for all conflict-free sets $E \subseteq A$ since $d_2 \notin E_F^+$ for every $E \in cf((A, R))$. It follows that $S_{X'} \cup \{\bar{\varphi}\}$ is cl-semi-stable. In a similar way we show that $S_{X'} \cup \{\varphi\}$ is cl-semi-stable in CF. Let $E = X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\} \cup \{\varphi\}$ for some $Z' \subseteq Z$ and $Y' \subseteq Y$ with $y_0 \in Y'$. Then E defends φ as $y_0 \in cl_i$ for all $i \leq n$, thus E is admissible. Moreover, $claim(E) = S_{X'} \cup \{\varphi\}$ and $\nu_{CF}(E) = \{d_a \mid a \in X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\}\} \cup \{\bar{\varphi}\}$. Similar as before we conclude that $claim(E) \cup \nu_{CF}(E)$ is subset-maximal among admissible claim-sets. □

We are now in the position to prove the desired Π_3^P-hardness result.

Proposition 18. *Con^{CAF}_{sem} is Π^{P}_3-hard.*

Proof. Let $CF = (A, R, claim)$ be the CAF generated by Reduction 7 from the given $QSAT^{\exists}_3$ instance $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ and let $F = (A, R)$. We show that Ψ is valid iff $sem_c(CF) \neq cl\text{-}sem(CF)$. Since $sem_c(CF) \subseteq cl\text{-}sem(CF)$ by Lemma 9, the latter reduces to showing that $sem_c(CF)$ is a proper subset of $cl\text{-}sem(CF)$, that is, we show that Ψ is valid iff there is some $X' \subseteq X$ such that $S_{X'} \cup \{\bar{\varphi}\}$ is not sem_c-realizable in CF.

Let us first assume that Ψ is valid, that is, there is $X' \subseteq X$ such that for all $Y' \subseteq Y$, there is $Z' \subseteq Z$ such that $X' \cup Y' \cup Z'$ is a model of φ. We show $S_{X'} \cup \{\bar{\varphi}\} \notin sem_c(CF)$. Towards a contradiction, assume there is $E \in sem((A, R))$ with $claim(E) = S_{X'} \cup \{\bar{\varphi}\}$. Then $\bar{\varphi} \in E$. By Lemma 8, we have $E^{\oplus}_F = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_1, d_2\})$. Let $Y' = E \cap Y$. By assumption Ψ is valid, there is $Z' \subseteq Z$ such that $M = X' \cup Y' \cup Z'$ is a model of φ. Let $D = M \cup \{\bar{v} \mid v \notin M\} \cup \{\varphi\}$. D is conflict-free; moreover, D attacks every cl_i, $i \leq n$, using that M is a model of φ: For each clause cl_i, there is $v \in V$ such that either $v \in cl_i \cap M$ (in that case, $v \in D$ and $(v, cl_i) \in R$) or $\bar{v} \in cl_i$ and $v \notin M$ (in that case, $\bar{v} \in D$ and $(\bar{v}, cl_i) \in R$). It follows that D is admissible as φ is defended against each attack of clause-arguments cl_i. Next we show that $D^{\oplus}_F$ is a proper superset of $E^{\oplus}_F$: Clearly, $V \cup \bar{V} \subseteq D^{\oplus}_F$; also, $\mathcal{C} \subseteq D^{\oplus}_F$ as shown above; moreover, $\bar{\varphi}, d_1 \in D^{\oplus}_F$ since $\varphi \in D$. As D and E contain the same arguments $a \in X \cup \bar{X} \cup Y \cup \bar{Y}$ by construction, we furthermore have $\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} = \{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus D\}$. It follows that $D^{\oplus}_F = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_2\})$. Thus D is admissible and $D^{\oplus}_F \supset E^{\oplus}_F$, contradiction to our assumption E is semi-stable in (A, R).

Next assume Ψ is not valid. We show that for all $X' \subseteq X$, $S_{X'} \cup \{\bar{\varphi}\} \in sem_c(CF)$. Fix $X' \subseteq X$. Since Ψ is not valid, there is $Y' \subseteq Y$ such that for all $Z' \subseteq Z$, $X' \cup Y' \cup Z'$ is not a model of φ. Fix $Z' \subseteq Z$ and let $E = X' \cup Y' \cup Z' \cup \{\bar{v} \mid v \notin X' \cup Y' \cup Z'\} \cup \mathcal{C}' \cup \{\bar{\varphi}\}$, where $\mathcal{C}' \subseteq \mathcal{C}$ contains all clauses cl_i which are not attacked by $X' \cup Y' \cup Z' \cup \{\bar{a} \mid a \notin X' \cup Y' \cup Z'\}$. Then E is admissible and $E^{\oplus}_F = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_1, d_2\})$. We show that E is semi-stable in (A, R): Assume there is $D \subseteq A$ with $D^{\oplus}_F \supset E^{\oplus}_F$. First observe that D attacks the same arguments d_a, $a \in X \cup \bar{X} \cup Y \cup \bar{Y}$, as E and thus $X' \cup Y' \subseteq D$. By Lemma 8 and since $D^{\oplus}_F$ is strictly bigger than $E^{\oplus}_F$, we have that $D^{\oplus}_F = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus D\} \cup \{d_2\})$. It follows that $\varphi \in D$. Let $Z'' = D \cap Z$. Then $M = X' \cup Y' \cup Z''$ is a model of φ: As each cl_i, $i \leq n$, is attacked by D, there is a literal $l \in D$ with $l \in cl_i$; now, if l is a positive literal, we have $l \in M$, in case l is a negative literal, we have $l \notin M$. Thus φ is satisfied by M, contradiction to our initial assumption Ψ is not valid. It follows that $S_{X'} \cup \{\bar{\varphi}\} \in sem_c(CF)$ for all $X' \subseteq X$. Thus $sem_c(CF) = cl\text{-}sem(CF)$ by Lemma 9. □

Π^{P}_3-hardness of Con^{CAF}_{stg} also uses Reduction 7 since $stg_c(CF) = sem_c(CF)$ and $cl\text{-}stg(CF) = cl\text{-}sem(CF)$ for all CAFs CF generated via the reduction. The proof proceeds similar as the proof of Lemma 9 and can be found in the appendix.

Lemma 10. *Let $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ be an instance of $QSAT^{\exists}_3$ and let $CF = (A, R, claim)$ be as in Reduction 7. Then*

1. *$cl\text{-}sem(CF) = cl\text{-}stg(CF)$; and*
2. *$sem_c(CF) = stg_c(CF)$.*

Π^{P}_3-hardness of Con^{CAF}_{stg} thus follows from the above lemma and from Proposition 18.

Proposition 19. *Con^{CAF}_{stg} is Π^{P}_3-hard.*

5.2 Concurrence of Well-formed CAFs

For well-formed CAFs, cl-preferred and i-preferred as well as all considered variants of stable semantics coincide [8] thus the respective problems become trivial. Since for semi-stable and stage semantics, the complexity for verification drops for both variants, we get the Π_2^{P}-membership results, by using the same generic membership argument as for general CAFs.

As coNP-hardness of deciding concurrence for naive semantics has been proven in [25] it remains to show matching hardness results for semi-stable and stage concurrence. This is by a reduction from $QSAT_2^{\forall}$ with some appropriate adaptions of Reduction 2.

Reduction 8. *Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT_2^{\forall}$, where φ is given by a set of clauses $C = \{cl_1, \dots, cl_n\}$ over atoms in $X = Y \cup Z$. Let (A, R) be the AF constructed from φ as in Reduction 2. We define $CF = (A', R', claim)$ with*

$$\begin{aligned} A' &= A \cup \{e, d_1, d_2, \bar{\varphi}_1, \bar{\varphi}_2\} \\ R' &= R \cup \{(a, d_a)(d_a, d_a) \mid a \in Y \cup \bar{Y}\} \cup \{(d_i, d_j) \mid i, j = 1, 2\} \cup \\ &\quad \{(a, b) \mid a, b \in \{\varphi, \bar{\varphi}_1, \bar{\varphi}_2\}, a \neq b\} \cup \{(\varphi, e), (e, e), (\varphi, d_1), (\bar{\varphi}_1, d_1)\} \end{aligned}$$

and $claim(d_1) = claim(d_2) = d$ and $claim(v) = v$ otherwise.

An example to illustrate the reduction is given in Figure 10. We observe that conflict-free claim-sets admit a close correspondence to their realizations in the underlying AF since all arguments except the self-attacking arguments d_1 and d_2 have been assigned unique claims. The following observations are easy to verify.

Lemma 11. *Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT_2^{\forall}$, let $\sigma \in \{sem, stg\}$ and let $CF = (A, R, claim)$ be as in Reduction 8. Then*

1. *for all $E \in cf((A, R))$, $(claim(E))^+_{CF} = E^+_{(A,R)} \setminus \{d_1\}$;*
2. *every $S \in cf_c(CF)$ admits a unique realization in (A, R);*
3. *for all $S \in \sigma_c(CF) \cup cl\text{-}\sigma(CF)$, either $\varphi \in S$ or $\varphi_1 \in S$ or $\varphi_2 \in S$.*

The following two lemmas will be useful to prove Π_2^{P}-hardness of Con_σ^{wf} for semi-stable and stage semantics. First, we will show that each inherited semi-stable (i-stage) claim-set is cl-semi-stable (cl-stage).

Lemma 12. *Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT_2^{\forall}$, let $\sigma \in \{sem, stg\}$ and let $CF = (A, R, claim)$ be as in Reduction 8. Then $\sigma_c(CF) \subseteq cl\text{-}\sigma(CF)$.*

Proof. Let $F = (A, R)$ and consider $S \in \sigma_c(CF)$ and let E denote the unique σ_c-realization of S in (A, R). As $E \in \sigma((A, R))$, we have that $E \cup E_F^+$ is subset-maximal among admissible (conflict-free) extensions. We will show that $S \cup S_{CF}^+$ is subset-maximal among i-admissible (i-conflict-free) claim-sets. Towards a contradiction, assume $S \cup S_{CF}^+$ is not subset-maximal among i-admissible (i-conflict-free) claim-sets, that is, there is $T \in adm_c(CF)$ ($T \in cf_c(CF)$) with $T \cup T_{CF}^+ \supset S \cup S_{CF}^+$. Consider the unique cf_c-realization D of T in (A, R), then $D \cup D_F^+ \setminus \{d_1\} = T \cup T_{CF}^+ \supset S \cup S_{CF}^+ = E \cup E_F^+ \setminus \{d_1\}$. If either $d_1 \in D_F^+$ or $d_1 \notin E_F^+$ we are done since in this case, we have $D \cup D_F^+ \supset E \cup E_F^+$, contradiction to E being semi-stable (stage) in (A, R). Thus we assume $d_1 \in E_F^+$ but $d_1 \notin D_F^+$. By Lemma 11, we have $\varphi_2 \in D$ since φ_2 does not attack d_1; also, $\varphi_1 \in E$ or $\varphi \in E$. In case $\varphi \in E$, we have $e \in E_F^+$, $e \notin D_F^+$ thus $e \in S \cup S_{CF}^+$ but $e \notin T \cup T_{CF}^+$, contradiction to the assumption $T \cup T_{CF}^+ \supset S \cup S_{CF}^+$. In case $\varphi_2 \in D$ and $\varphi_1 \in E$, consider $D' = (D \cup \{\varphi_1\}) \setminus \{\varphi_2\}$. D' is admissible (conflict-free) as D is admissible (conflict-free) and exchanging φ_2 with φ_1 does neither add conflicts nor undefended arguments. Moreover, $d_1 \in (D')_F^+$ and $D \cup D_F^+ = D' \cup (D')_F^+ \setminus \{d_1\}$. Therefore $D' \cup (D')_F^+ \supset E \cup E_F^+$, contradiction to E being semi-stable (stage) in (A, R). □

Next we will prove that each semi-stable (stage) claim-set that contains φ is both inherited and claim-level semi-stable (stage).

Lemma 13. *Let $\Psi = \forall Y \exists Z \varphi(Y,Z)$ be an instance of $QSAT_2^{\forall}$, let $\sigma \in \{sem, stg\}$ and let $CF = (A, R, claim)$ be as in Reduction 8. Then for all $S \in \sigma_c(CF) \cup cl\text{-}\sigma(CF)$, $\varphi \in S$ implies $S \in \sigma_c(CF) \cap cl\text{-}\sigma(CF)$.*

Proof. Let $F = (A, R)$. By Lemma 12, $\sigma_c(CF) \subseteq cl\text{-}\sigma(CF)$ thus it suffices to prove the statement for $S \in cl\text{-}\sigma(CF)$. Let E denote the unique cf_c-realization of S in (A, R). We will show $E \in \sigma((A, R))$. Towards a contradiction, assume there is $D \in adm((A, R))$ ($D \in cf((A, R))$) with $D \cup D_F^+ \supset E \cup E_F^+$. As $\varphi \in E$ we have $d_1 \in E_F^+$ and thus $D \cup D_F^+ \setminus \{d_1\} \supset E \cup E_F^+ \setminus \{d_1\}$. By Lemma 11, $claim(D) \cup claim(D)_F^+ = D \cup D_F^+ \setminus \{d_1\} \supset E \cup E_F^+ \setminus \{d_1\} = S \cup S_{CF}^+$, contradiction to S being cl-semi-stable (cl-stage) in CF. □

Proposition 20. *Con_{σ}^{wf}, $\sigma \in \{sem, stg\}$, is Π_2^{P}-hard.*

Proof. Let $\Psi = \forall Y \exists Z \varphi(Y,Z)$ be an instance of $QSAT_2^{\forall}$ and let $CF = (A, R, claim)$ be as in Reduction 8. Moreover, let $F = (A, R)$.

We will show Ψ is valid iff $\sigma_c(CF) = cl\text{-}\sigma(CF)$.

First assume Ψ is valid. We show that in this case, $\varphi \in S$ for all $S \in \sigma_c(CF) \cup cl\text{-}\sigma(CF)$. By Lemma 13, this implies $S \in \sigma_c(CF) \cap cl\text{-}\sigma(CF)$ and thus $\sigma_c(CF) = cl\text{-}\sigma(CF)$.

By Lemma 12, it suffices to prove the statement for every $S \in cl\text{-}\sigma(CF)$. Towards a contradiction, assume there is $S \in cl\text{-}\sigma(CF)$ such that $\varphi \notin S$. Then $e \notin S \cup S_{CF}^+$. Let $Y' = S \cap Y$. Since Ψ is valid, there is $Z' \subseteq Z$ such that $Y' \cup Z'$ is a model of φ. Let $E = Y' \cup Z' \cup \{\bar{x} \mid x \notin Y' \cup Z'\} \cup \{\varphi\}$. Then $S' = claim(E)$ is i-admissible (i-conflict-free) and $S' \cup (S')_{CF}^+ = claim(A) \setminus (\{d\} \cup \{d_y \mid y \notin E\} \cup \{d_{\bar{y}} \mid \bar{y} \notin E\})$. We can conclude that $S' \cup (S')_{CF}^+ \supset S \cup S_{CF}^+$ since $e \notin S \cup S_{CF}^+$ and $\{d\} \cup \{d_y \mid y \notin E\} \cup \{d_{\bar{y}} \mid \bar{y} \notin E\} \nsubseteq S \cup S_{CF}^+$, contradiction to our initial assumption S is cl-semi-stable (cl-stage). It follows that $\varphi \in S$ for every $S \in cl\text{-}\sigma(CF)$.

Now assume Ψ is not valid, i.e., there is $Y' \subseteq Y$ such that for all $Z' \subseteq Z$, $Y' \cup Z'$ is not a model of φ. We will show that $\sigma_c(CF) \subset cl\text{-}\sigma(CF)$. Fix $Z' \subseteq Z$ and let $E = Y' \cup Z' \cup \{\bar{x} \mid x \notin Y' \cup Z'\}$. Moreover, let $E_1 = E \cup \mathcal{C}' \cup \{\varphi_1\}$ and $E_2 = E \cup \mathcal{C}' \cup \{\varphi_2\}$ where $\mathcal{C}' \subseteq \mathcal{C}$ contains all clauses cl_i such that $E \cap cl_i = \emptyset$. Clearly, $E_1, E_2 \in adm((A, R))$ ($E_1, E_2 \in cf((A, R))$) and thus $E_1 = claim(E_1), E_2 = claim(E_2) \in adm_c(CF)$ ($E_1 = claim(E_1), E_2 = claim(E_2) \in cf_c(CF)$). Observe that $(E_2)_F^{\oplus} \subset (E_1)_F^{\oplus}$ since d_1 is attacked by $\varphi_1 \in E_1$ but there is no $a \in E_2$ such that $(a, d_1) \in R$. It follows that $E_2 = claim(E_2) \notin \sigma_c(CF)$. We show that $E_2 \in cl\text{-}\sigma(CF)$ for $\sigma \in \{sem, stg\}$, that is, we show that $claim(E_2) \cup (E_2)_{CF}^+ = claim(A) \setminus (\{e, d\} \cup \{d_y \mid y \notin E\} \cup \{d_{\bar{y}} \mid \bar{y} \notin E\})$ is maximal among admissible (conflict-free) claim-sets: Towards a contradiction, assume there is $T \in adm_c(CF)$ ($T \in cf_c(CF)$) such that $T \cup T_{CF}^+ \supset claim(E_2) \cup (E_2)_{CF}^+$. As $\{d_y \mid y \in Y'\} \cup \{d_{\bar{y}} \mid y \notin Y'\} \subseteq T_{CF}^+$ we have $Y' \cup \{\bar{y} \mid y \notin Y'\} \subseteq T$ and T_{CF}^+ does not contain any claim in $\{d_y \mid y \notin E\} \cup \{d_{\bar{y}} \mid \bar{y} \notin E\}$ since for every $y \in Y$, there is no conflict-free set attacking both d_y and $d_{\bar{y}}$. Moreover, $d \notin T_{CF}^+$ for every $T \in cf_c(CF)$ since d_1 and d_2 are the only attackers of d_2 and d_1 is self-attacking. It follows that $e \in T_{CF}^+$ and thus $\varphi \in T$. Consider the unique cf_c-realization D of T. Since $\varphi \in D$ we have we have $cl_i \notin D$ for every $i \leq n$ and thus each cl_i is attacked by D. Let $M = D \cap X$ and consider an arbitrary clause cl_i. As each cl_i is attacked by D, there is either $x \in D$ with $x \in cl_i$ or $\bar{x} \in D$ with $\bar{x} \in cl_i$. In the former case, we have $x \in M$ and thus cl_i is satisfied, in the latter case, $x \notin M$ and thus cl_i is satisfied. Thus M is a model of φ and $Y' \subseteq M$, contradiction to our initial assumption $Y' \cup Z''$ is not a model of φ for every $Z'' \subseteq Z$. □

5.3 Coherence and Concurrence of Stable Variants

We conclude this section by analyzing two related problems. First, we ask ourselves how hard it is to decided whether the two variants of the claim-based stable semantics coincide. Bearing in mind the complexity of the verification problem of the two semantics, the problem has to be contained in Π_2^{P}; however, as we show next, it is also hard for this class for general CAFs. For well-formed CAFs recall that the two variants collapse anyway making this problem trivial for well-formed CAFs.

Proposition 21. *Given a CAF $CF = (A, R, claim)$, deciding whether $cl\text{-}stb_{cf}(CF) = cl\text{-}stb_{adm}(CF)$ is Π_2^{P}-complete.*

Proof. We present a Σ_2^{P}-procedure for the complementary problem.

(1) Guess a set $S \subseteq claim(A)$;

(2) check $S \in cl\text{-}stb_{cf}(CF)$ and $S \notin cl\text{-}stb_{adm}(CF)$.

The latter can be checked in NP, respectively, coNP.

We present a reduction from $QSAT_2^{\forall}$. Let $\Psi = \forall Y \exists Z \varphi(Y, Z)$ be an instance of $QSAT_2^{\forall}$, where φ is given by a set of clauses $C = \{cl_1, \dots, cl_n\}$ over atoms in $X = Y \cup Z$. We construct a CAF $CF = (A, R, claim)$ given by

- $A = X \cup \bar{X} \cup \mathcal{C} \cup \{\varphi, \bar{\varphi}\}$;
- $R = \{(x, cl_i) \mid x \in cl_i\} \cup \{(\bar{x}, cl_i \mid \bar{x} \in cl_i\} \cup \{(cl_i, cl_i), (cl_i, \varphi) \mid i \leq n\} \cup \{(x, \bar{x}), (\bar{x}, x) \mid x \in X\} \cup \{(\varphi, \bar{\varphi}), (\bar{\varphi}, \bar{\varphi})\} \cup \{(\bar{\varphi}, z) \mid z \in Z\} \cup \{(\bar{\varphi}, \bar{z}) \mid \bar{z} \in \bar{Z}\}$;
- $claim(y) = y$, $claim(\bar{y}) = \bar{y}$ for $y \in Y$, $\bar{y} \in \bar{Y}$, $claim(z) = claim(\bar{z}) = claim(cl_i) = claim(\varphi) = claim(\bar{\varphi}) = c$ for $i \leq n$, $z \in Z$, $\bar{z} \in \bar{Z}$.

See Figure 11 for an illustrative example. We show

(a) for all $Y' \subseteq Y$, $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \in cl\text{-}stb_{cf}(CF)$. Moreover, there is no other cf-cl-stable claim-set in CF.

Let $Y' \subseteq Y$ be arbitrary, let $z \in Z$ and let $E = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{z\}$. Clearly, E is conflict-free in (A, R); moreover, E attacks every $a \in A$ such that $claim(a) \notin claim(E)$. It follows that $claim(E) = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \in cf_c(CF)$. Moreover, $claim(E)$ is maximal among all conflict-free claim-sets: Assume there is $T \in cf_c(CF)$ such that $T \supset claim(E)$ for some $Y' \subseteq Y$. Then there is $y \in Y$ such that $y \in T$ and $\bar{y} \in T$, contradiction to cf-realizability of T since for every $y \in Y$, y and $\bar{y}$ mutually attack each other. We can furthermore conclude that no other cl-stable claim-set exists since for every $y \in Y$, y and $\bar{y}$ mutually attack each other. Thus each cf-cl-stable claim-set is of the form $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\}$ for some $Y' \subseteq Y$.

(b) Ψ is valid iff $cl\text{-}stb_{adm}(CF) = cl\text{-}stb_{cf}(CF)$.

Assume Ψ is valid. We show that $stb_c(CF) = cl\text{-}stb_{cf}(CF)$, $cl\text{-}stb_{adm}(CF) = cl\text{-}stb_{cf}(CF)$ then follows since $stb_c(CF) \subseteq cl\text{-}stb_{adm}(CF) \subseteq cl\text{-}stb_{cf}(CF)$. Let $Y' \subseteq Y$. Then there is $Z' \subseteq Z$ such that φ is satisfied by $M = Y' \cup Z'$. Let $E = M \cup \{\bar{x} \mid x \notin M\} \cup \{\varphi\}$. Since M satisfies each clause cl_i, there is either $x \in cl_i$ with $x \in M$ or there is $\bar{x} \in cl_i$ with $x \notin M$. It follows that each cl_i, $i \leq n$, is attacked by E; moreover, E attacks $\bar{\varphi}$ since $\varphi \in E$. Since E is also conflict-free we have shown that E is a stable extension of (A, R) and therefore $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \in stb_c(CF)$. As Y' was arbitrary, we have that $Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \in stb_c(CF)$ for all $Y' \subseteq Y$. We conclude that $stb_c(CF) = cl\text{-}stb_{adm}(CF) = cl\text{-}stb_{cf}(CF)$ by (a).

Assume $cl\text{-}stb_{adm}(CF) = cl\text{-}stb_{cf}(CF)$. Let $Y' \subseteq Y$. By (a) we have that $S = Y' \cup \{\bar{y} \mid y \notin Y'\} \cup \{c\} \in cl\text{-}stb_{adm}(CF) = cl\text{-}stb_{cf}(CF)$. Consider an *adm*-realization E of S and let $Z' = E \cap Z$. Then $M = Y' \cup Z'$ satisfies φ: First observe that $\varphi \in E$: Since $c \in S$, there is some $a \in A$ with $claim(a) = c$ such that $a \in E$. Moreover, $a \in Z \cup \bar{Z} \cup \{\varphi\}$ since every other claim assigned with c is self-attacking. In case $a = \varphi$, we are done; in case $a = z$ or $a = \bar{z}$ for some $z \in Z$ we have $\varphi \in E$ since E defends a against $\bar{\varphi}$. Since $\varphi \in E$, we furthermore have that E attacks each clause cl_i since φ is defended by E against cl_i. Now, consider an arbitrary clause cl_i. As E attacks cl_i there is either an argument $x \in E$ with $(x, cl_i) \in R$ or an argument $\bar{x} \in E$ with $(\bar{x}, cl_i) \in R$. In the former case, $x \in cl_i$ and $x \in M$ and thus cl_i is satisfied ; in the latter case, $\bar{x} \in cl_i$ and $x \notin M$ and thus cl_i is satisfied. Thus M is a model of φ. We have shown that for every $Y' \subseteq Y$, there is $Z' \subseteq Z$ such that $Y' \cup Z'$ satisfies φ. It follows that Ψ is valid.

□

The second problem we would like to discuss here is the well-known coherence problems, which asks whether for a given AF its preferred and stable extensions coincide, shown Π_2^{P}-complete in [23]. The problem was studied for inherited semantics in [12] showing that complexity remains on the second level. The forthcoming result shows that, although the complexity of the verification task increases for claim-based preferred semantics, testing coherence for CAFs in terms of cl-semantics is of the same complexity as in the AF setting, as well.

Proposition 22. *Given a CAF $CF = (A, R, claim)$, $\sigma \in \{cf, adm\}$ deciding whether $cl\text{-}stb_\sigma(CF) = cl\text{-}prf(CF)$ is Π_2^{P}-complete; hardness holds even for well-formed CAFs.*

Proof. We present a Σ_2^{P}-procedure for the complementary problem.

(1) Guess a set $S \subseteq claim(A)$;

(2) check $S \in (cl\text{-}stb_\sigma(CF) \setminus cl\text{-}prf(CF)) \cup (cl\text{-}prf(CF) \setminus cl\text{-}stb_\sigma(CF))$.

Verifying that S is cl-preferred is DP-complete, verifying that S is cl-stable is NP-complete, yielding a Σ_2^{P}-algorithm.

Hardness follows from the corresponding result for AFs, i.e., deciding coherence for AFs is Π_2^{P}-complete. □

6 Tractable Fragments

While most of the decision problems considered in Section 4 are intractable, some of them become tractable when restricted to specific graph classes or when parameterized by some criterion characterizing the structure of the framework. Thus, in what follows, we will revisit those decision problems and investigate their complexities when restricted to such graph classes or when parameterized by the number of claims within the framework. This is in the line of similar investigations for AFs where tractable graph classes have been considered [26, 6] as well as fixed-parameter tractable algorithms [27, 28, 29].

6.1 Graph classes

We will consider five graph classes that have proven themselves promising for acquiring improved bounds for Dung AFs [26, 6]. Based on their graph structure, we will consider CAFs $CF = (A, R, claim)$ that fall into one of these five classes:

- *Acyclic CAFs*, if there is no directed cycle in (A, R).

- *Noeven CAFs*, if there is no directed cycle of even length in (A, R).
- *Symmetric CAFs*, if the attack relation R is symmetric, i.e. whenever $(a, b) \in R$ then also $(b, a) \in R$.
- *Symmetric irreflexive CAFs*, if CF is symmetric and contains so self-attacks, i.e. $(a, a) \notin R$ for all $a \in A$.
- *Bipartite CAFs*, if (A, R) is a bipartite graph, i.e. does not contain an undirected cycle of even length.

We recall that on well-formed CAFs, the inherited and claim-level variants coincide for preferred and stable semantics. Thus for cl-preferred and cl-stable semantics in well-formed CAFs, the complexity results for credulous and skeptical reasoning as well as verification carry over from the respective inherited counterparts [12].

6.1.1 Acyclic CAFs

For acyclic CAFs, we obtain tractability for most of the considered problems since all considered admissible-based as well as all range-based semantics coincide with grounded semantics. This is an immediate consequence of the respective property for acyclic AFs where $grd(F) = prf(F) = stb(F) = sem(F) = stg(F)$ for each acyclic AF F [15].

Proposition 23. *For acyclic CAFs, for* $\Delta \in \{CAF, wf\}$*,* $Cred_\sigma^\Delta$*,* $Skept_\sigma^\Delta$*, and* Ver_σ^Δ *is in* P *for* $\sigma \in \{cl\text{-}prf, cl\text{-}stb_{cf}, cl\text{-}stb_{adm}, cl\text{-}sem, sem_c, cl\text{-}stg, stg_c\}$.

For cl-naive semantics, on the other hand, the restriction to acyclic graphs does not yield any computational advantages. To obtain Π_2^P-hardness for skeptical acceptance and DP-hardness for verification in the general case, we adapt Reduction 1 by taking unidirectional instead of bidirectional edges; acyclicity can be easily guaranteed if e.g., each argument which corresponds to a positive atom has only outgoing attacks and each argument corresponding to a negated atom has only incoming attacks; additionally, we remove all attacks from the argument φ. For coNP-hardness of skeptical acceptance for cl-naive semantics for well-formed CAFs, we adapt the reduction from the proof of Proposition 14 accordingly, e.g., by removing all attacks from arguments representing positive literals. We thus obtain the following result.

Proposition 24. *For acyclic CAFs,* $Cred_{cl\text{-}naive}^\Delta$*,* $\Delta \in \{CAF, wf\}$*, and* $Ver_{cl\text{-}naive}^{wf}$ *is in* P*;* $Skept_{cl\text{-}naive}^{CAF}$ *is* Π_2^P*-complete;* $Skept_{cl\text{-}naive}^{wf}$ *is* coNP*-complete; and* $Ver_{cl\text{-}naive}^{CAF}$ *is* DP*-complete.*

We note that NE_σ^Δ, $\Delta \in \{CAF, wf\}$ is trivial for all considered semantics σ since the grounded extension is non-empty (assuming $A \neq \emptyset$).

6.1.2 Noeven CAFs

We first recall that grounded, preferred, and semi-stable semantics coincide for each noeven AF $F = (A, R)$, and $grd(F) = stb(F)$ if $grd(F) \neq \{\emptyset\}$ [30, 15]. We thus obtain that $grd_c(CF) = cl\text{-}sem(CF) = sem_c(CF) = prf_c(CF)$, moreover, $stb_c(CF) = cl\text{-}stb_{adm}(CF)$ since the underlying AF has a unique preferred extension that serves as candidate set for realizing a stable claim-set. Since the grounded extension can be computed in P we obtain the following results.

Proposition 25. *For noeven CAFs, for* $\Delta \in \{CAF, wf\}$*,* $Cred_\sigma^\Delta$*,* $Skept_\sigma^\Delta$*,* Ver_σ^Δ*, and* NE_σ^Δ *is in* P *for* $\sigma \in \{cl\text{-}prf, cl\text{-}stb_{adm}, cl\text{-}sem, sem_c\}$.

Proposition 24 also applies in the noeven case.

Proposition 26. *For noeven CAFs, $Cred^{\Delta}_{cl\text{-}naive}$, $\Delta \in \{CAF, wf\}$, and $Ver^{wf}_{cl\text{-}naive}$ is in* P*; $Skept^{CAF}_{cl\text{-}naive}$ is Π^{P}_2-complete; $Skept^{wf}_{cl\text{-}naive}$ is* coNP*-complete; and $Ver^{CAF}_{cl\text{-}naive}$ is* DP*-complete.*

However, for the *cl-stb$_{cf}$* semantics, the problems remain hard. Towards this, we introduce the following reduction

Reduction 9. *Let φ be an instance of 3-SAT, with φ given as a set of clauses $C = \{c_1, \dots, c_n\}$ over atoms in X, where negated atoms are denoted by $\bar{x}$. We construct $CAF_\varphi = (A, R, claim)$ with*

$$
\begin{aligned}
A &= X \cup \bar{X} \cup C \\
R &= \{(x, \bar{x}) \mid x \in X\} \cup \{(l, c) \mid c \in C, l \in c\} \cup \{(c, c) \mid c \in C\}
\end{aligned}
$$

with $claim(x) = claim(\bar{x}) = \psi$ for all $x \in X$ and $claim(c) = c$ for all $c \in C$. An illustrative example of the reduction is given in Figure 12. Note, that the only directed cycles contained in CAF_φ are the self-attacks of the arguments in C, thus CAF_φ is noeven.

Proposition 27. *For noeven CAFs, $Cred^{CAF}_{cl\text{-}stb_{cf}}$, $Ver^{CAF}_{cl\text{-}stb_{cf}}$, and $NE^{CAF}_{cl\text{-}stb_{cf}}$ are* NP*-complete; $Skept^{CAF}_{cl\text{-}stb_{cf}}$ is* coNP*-complete.*

Proof. Upper bounds are obtained via the case for general CAFs, c.f. Table 4. For the lower bounds, we start with the NP-complete problems.

For a given instance of 3-SAT φ, we construct a CAF_φ as in Reduction 9. Note, that the arguments $c \in C$ are all self-attacking and thus can never be part of any conflict-free set of arguments of the Dung AF underlying CAF_φ. Therefore, their claims cannot be part of any *cl-stb$_{cf}$* extension of CAF_φ. Furthermore, trivially, $\emptyset$ cannot be a *cl-stb$_{cf}$* extension of CAF_φ either. Thus, the only candidate *cl-stb$_{cf}$* extension of CAF_φ is $\{\psi\}$ and therefore, $Cred^{CAF}_{cl\text{-}stb_{cf}}(CAF_\varphi, \psi) = Ver^{CAF}_{cl\text{-}stb_{cf}}(CAF_\varphi, \{\psi\}) = NE^{CAF}_{cl\text{-}stb_{cf}}(CAF_\varphi)$. We will show that φ is satisfiable iff $Ver^{CAF}_{cl\text{-}stb_{cf}}(CAF_\varphi, \{\psi\})$.

First, assume that φ is satisfiable and let M be a model of φ. Then, the set $E = M \cup \overline{X \setminus M}$ is conflict-free in the underlying Dung AF of CAF_φ by the construction of CAF_φ. Furthermore, as all $c \in C$ are satisfied by M, there must be some $l \in E$ such that $(l, c) \in R$ for all $c \in C$. Thus, E attacks all arguments $c \in C$ and therefore $claim(E) \cup \nu_{CAF_\varphi}(E) = \{\psi\} \cup \{C\} = claim(A)$, making $\{\psi\}$ a *cl-stb$_{cf}$* extension of CAF_φ.

Now, assume that φ is unsatisfiable. Then, for every conflict-free set of arguments $E \subseteq X \cup \bar{X}$ in the underlying Dung AF of CAF_φ, there exists some $c \in C$ such that $(l, c) \notin R$ for all $l \in E$, as otherwise $E \cap X$ would be a model of φ by construction. Therefore, $c \notin \nu_{CAF_\varphi}(E)$ for some $c \in C$ and thus $\{\psi\}$ is not a *cl-stb$_{cf}$* extension of CAF_φ.

The result for the $Skept^{CAF}_{cl\text{-}stb_{cf}}$ problem can be proven similarly by reducing from 3-UNSAT while using the same construction CAF_φ as before, but with $claim(c) = \gamma$ for all $c \in C$ and without the self-attacks of the arguments in C. If φ is satisfiable, then $\{\psi\}$ is a *cl-stb$_{cf}$* extension of CAF_φ by an analogous argument as before and thus, γ is not skeptically accepted in CAF_φ w.r.t. the *cl-stb$_{cf}$* semantics. However, if φ is unsatisfiable, as before, $\{\psi\}$ cannot be a *cl-stb$_{cf}$* extension of CAF_φ, as otherwise φ would be satisfiable and thus, γ is skeptically accepted in CAF_φ w.r.t. the *cl-stb$_{cf}$* semantics, as $\emptyset$ is trivially not a *cl-stb$_{cf}$* extension of CAF_φ and all other possible extension contain γ. □

Next, we look at the semantics based on the stage and semi-stable semantics.

Proposition 28. *For noeven CAFs, for $\Delta \in \{CAF, wf\}$, $Cred_\sigma^\Delta$ is Σ_2^P-complete and $Skept_\sigma^\Delta$ is Π_2^P-complete for $\sigma \in \{stg_c, cl\text{-}stg\}$.*

Proof. We obtain upper bounds from the corresponding problems for general CAFs. Lower bounds can be obtained via the respective results for noeven Dung AFs [15], which carry over to CAFs by assigning every argument a unique claim. □

Next we turn to the verification problem for noeven well-formed CAFs with respect to stage semantics. We obtain coNP-membership from well-formed CAFs (cf. Table 6). For hardness, we show that this problem is already intractable for noeven Dung AFs.

We make use of the following reduction.

Reduction 10. *Let φ be an instance of 3-UNSAT, with φ given as a set of clauses $C = \{c_1, \dots, c_n\}$ over atoms in X, where negated atoms are denoted by $\bar{x}$. We construct $AF_\varphi = (A, R)$ with*

$$\begin{aligned} A &= X \cup \bar{X} \cup C \cup \{y\} \\ R &= \{(x, \bar{x}) \mid x \in X\} \cup \{(l, c) \mid c \in C, l \in c\} \cup \{(c, c) \mid c \in C\} \cup \\ &\quad \{(x, y), (\bar{x}, y) \mid x \in X\} \cup \{(y, c) \mid c \in C\} \end{aligned}$$

for a fresh atom y. An illustrative example of the reduction is given in Figure 13. Note that the only directed cycles contained in AF_φ are the self-attacks of the arguments in C, thus AF_φ is noeven.

Proposition 29. *Ver_{stg}^F is coNP-complete for noeven Dung AFs.*

Proof. The upper bound can be obtained from the case for Dung AFs in general [15]. We show the lower bound via reduction from 3-UNSAT. Let φ be an instance of 3-UNSAT and $AF_\varphi = (A, R)$ be as in Reduction 10. We show that $\{y\}$ is a stage extension of AF_φ iff φ is unsatisfiable. To increase readability, we will omit the φ in the subscript for the remainder of this proof and just write AF instead of AF_φ. Note that the argument y is conflicting with every other argument, as y attacks all arguments $c \in C$ and is attacked by all arguments $x, \bar{x} \in X$. Thus, the only candidate stage extension containing y is the one containing only y, which has range $\{y\}_{AF}^+ = C \cup \{y\}$.
First, assume that φ is satisfiable and let M be a model of φ. Then, the set $E = M \cup \overline{X \setminus M}$ is conflict-free in AF by the construction of AF. Furthermore, as all $c \in C$ are satisfied by M, there must be some $l \in E$ such that $(l, c) \in R$ for all $c \in C$. Thus $E_{AF}^+ = M \cup \overline{X \setminus M} \cup C \cup \{y\} \supset C \cup \{y\} = \{y\}_{AF}^+$ and therefore $\{y\}$ is not a stage extension of AF.
Now, assume that φ is unsatisfiable. Then, for every conflict-free set of arguments $E \subset X \cup \bar{X}$, $c \notin E_{AF}^+$ for some $c \in C$, as otherwise $E \cap X$ would be a model of φ. Therefore, $\{y\}_{AF}^+ = C \cup \{y\}$ is maximal (with regard to $\subseteq$) in AF and thus $\{y\}$ is a stage extension of AF. □

As a consequence, we obtain coNP-completeness for the respective problem for noeven well-formed CAFs.

Proposition 30. *For noeven well-formed CAFs, Ver_σ^{wf} is coNP-complete for $\sigma \in \{stg_c, cl\text{-}stg\}$.*

Proof. Upper bounds are obtained from the case for CAFs in general [31]. Lower bounds generalize from the case for Dung AFs, c.f. Proposition 29, which carry over to well-formed CAFs by assigning every argument an unique claim. □

Proposition 31. *Ver_σ^{CAF} is Σ_2^P-complete for $\sigma \in \{stg_c, cl\text{-}stg\}$ for noeven CAFs.*

Proof. We present the proof for $\sigma = stg_c$, the proof for $\sigma = cl\text{-}stg$ is analogous. Upper bound via the general case for CAFs, lower bound via a reduction from the $Cred_{stg}$ problem for noeven Dung AFs. The $Cred_{stg}$ problem for noeven Dung AFs is known to be Σ_2^{P}-c [15]. To decide the problem for an argument b in an noeven Dung $AF = (A, R)$, construct a $CAF = (A' = A \cup \{x\}, R, claim)$ with a new argument $x \notin A$ and $claim(b) = c_1$ and $claim(a) = c_2$ for all $a \in A' \setminus \{b\}$. Then, argument b is credulously accepted in AF with regard to the stage semantics iff $\{c_1, c_2\}$ is a i-stage extension of CAF. □

Proposition 32. *For noeven CAFs, NE_σ^Δ, $\Delta \in \{CAF, wf\}$ is in* P *for $\sigma \in \{cl\text{-}naive, cl\text{-}prf, cl\text{-}stb_{adm}, cl\text{-}sem, sem_c, cl\text{-}stg, stg_c\}$.*

Proof. In order to decide non-emptiness for $\sigma \in \{cl\text{-}prf, cl\text{-}stb_{adm}, cl\text{-}sem\}$ it suffices to check whether there exists some unattacked argument. For cl-naive, i-naive, cl-stage, and i-stage semantics, it suffices to check whether there is some argument $a \in A$ that does not attack itself. □

6.1.3 Symmetric CAFs

For symmetric AFs, each conflict-free set defends itself, i.e., $cf(F) = adm(F)$ for each symmetric AF F. As an immediate consequence we obtain that each admissible-based semantics coincide with their conflict-free-based counterpart.

Lemma 14. *For each symmetric CAF CF, $cl\text{-}prf(CF) = cl\text{-}naive(CF)$, $stb_{cf}(CF) = cl\text{-}stb_{adm}(CF)$, $cl\text{-}sem(CF) = cl\text{-}stg(CF)$.*

Hardness results for cl-naive semantics correspond to the results for the general case since Reduction 1 is indeed symmetric; the reduction from the proof of Proposition 14 can be adapted by adding the required attacks between the clause-arguments and the literal-arguments. By the above observation we moreover obtain the respective results for cl-preferred semantics.

Proposition 33. *For symmetric CAFs, $Cred_\sigma^\Delta$, $\Delta \in \{CAF, wf\}$, $Skept_\sigma^{wf}$ and Ver_σ^{wf} is in* P*; $Skept_\sigma^{CAF}$ is Π_2^{P}-complete; and Ver_σ^{CAF} is* DP*-complete for $\sigma \in \{cl\text{-}naive, cl\text{-}prf\}$.*

As shown in [12], deciding credulous acceptance w.r.t. stable semantics remains NP-hard for symmetric AFs; likewise, deciding skeptical acceptance w.r.t. stable semantics remains coNP-hard for symmetric AFs. By assigning each argument a unique claim, we thus obtain the respective results for cl-stable semantics. Moreover, we obtain NP-completeness of verifying cl-stable claim-sets for symmetric CAFs by appropriate adaption of Translation Tr_2. Note that for well-formed CAFs, verification is solvable in polynomial time (cf. Table 6).

Proposition 34. *For symmetric CAFs, for $\Delta \in \{CAF, wf\}$, $Cred_\sigma^\Delta$ is* NP*-complete and $Skept_\sigma^\Delta$ is* coNP*-complete; moreover, Ver_σ^{CAF} is* NP*-complete and Ver_σ^{wf} is in* P *for $\sigma \in \{cl\text{-}stb_{cf}, cl\text{-}stb_{adm}\}$.*

Proof. To prove NP-completeness of verifying cl-stable claim-sets for symmetric CAFs, we first observe that membership for Ver_σ^{CAF} is by the corresponding result for general CAFs. For hardness, we provide a reduction from $Ver_{stb_c}^{CAF}$ for symmetric CAFs: We adapt Translation Tr_2 by setting $Tr_2'(CF) = (A', R' \cup \{(b, a) \mid (a, b) \in R'\}, claim')$ for $Tr_2(CF) = (A', R', claim')$, i.e., we make all attacks symmetric. We obtain $stb_c(CF) = stb_c(Tr_2'(CF)) = cl\text{-}stb_\tau(Tr_2'(CF))$ for $\tau \in \{cf, adm\}$ for any symmetric CAF CF. Thus, for an instance, i.e., a CAF CF and a claim-set S of $Ver_{stb_c}^{CAF}$ for symmetric CAFs, it suffices to check whether $Tr_2'(CF)$ is cl-stable. □

For most of the considered decision problems, both versions of semi-stable and stage semantics for symmetric (well-formed) CAFs admit the same complexity as the respective problems for AFs (cf. [15]; the lower bound for verification is obtained by translating standard Dung AFs to symmetric Dung AFs in a way such that stage extensions are preserved [15, Lemma 14]), with the notable exception of verification for general CAFs which remains as hard as in the general case.

Proposition 35. *For symmetric CAFs, for $\Delta \in \{CAF, wf\}$, $Cred_\sigma^\Delta$ is Σ_2^{P}-complete; $Skept_\sigma^\Delta$ is Π_2^{P}-complete; Ver_σ^{CAF} is Σ_2^{P}-complete and Ver_σ^{wf} is coNP-complete for $\sigma \in \{$cl-sem, sem$_c$, cl-stg, stg$_c\}$.*

Proof. For $Cred_\sigma^\Delta$, $Skept_\sigma^\Delta$, and Ver_σ^{wf}, lower bounds are by the corresponding results for AFs [15]; upper bounds are by the respective results for general CAFs (cf. Tables 4 and 6).

To show hardness of Ver_σ^{CAF} we reduce from $Cred_\sigma^F$ for symmetric AFs (Σ_2^{P}-complete): Given an AF $F = (A, R)$ and an argument $b \in A$, we assign the claims $claim(b) = c_1$, $claim(a) = c_2$, $a \in A \setminus \{b\}$. It can be shown that the argument b is credulously accepted iff the set of claims $\{c_1, c_2\}$ is cl-semi-stable (cl-stage) in the corresponding CAF $(A, R, claim)$. □

For $\sigma \in \{$*cl-naive*, *cl-prf*, *cl-sem*, *sem$_c$*, *cl-stg*, *stg$_c$*$\}$, to decide NE_σ^Δ, $\Delta \in \{CAF, wf\}$, it suffices to check whether CF contains an argument that does not attack itself. For stable semantics, the problem remains NP-hard already for Dung AFs.

Proposition 36. *NE_{stb}^{AF} is NP-complete for symmetric AFs.*

Proof. Membership is by the corresponding result for general AFs. Hardness is by the following reduction from SAT: Given a CNF φ with clauses C over atoms in X. We denote $\neg x$ by $\bar{x}$. We construct $F = (A, R)$ with

$$\begin{aligned} A &= X \cup \bar{X} \cup C \\ R &= \{(x, \bar{x}), (\bar{x}, x) \mid x \in X\} \cup \{(l, c), (c, l) \mid c \in C, l \in c\} \cup \{(c, c) \mid c \in C\} \end{aligned}$$

We show that $stb(F) \neq \emptyset$ iff φ is satisfiable. First, let $stb(F) \neq \emptyset$ and let $E \in stb(F)$. Clearly, $M = E \cap X$ is a model of φ since each clause is satisfied: Let $c \in C$, then there is $l \in E$ st l attacks c. In case l is a positive literal l is contained in M, in case l is a negative literal, we have l is not contained in M and thus c is satisfied in both cases. For the other direction, assume φ has a model M. Then $E = M \cup \{\bar{x} \mid x \notin M\}$ is a stable extension of F since each clause argument is attacked: As all $c \in C$ are satisfied by M, there must be some $l \in E$ such that $(l, c) \in R$ for all $c \in C$ by construction. □

We thus obtain the following result.

Proposition 37. *For symmetric CAFs, for $\Delta \in \{CAF, wf\}$, NE_σ^Δ is in P for $\sigma \in \{$cl-naive, cl-prf, cl-sem, sem$_c$, cl-stg, stg$_c\}$ and NP-complete for $\sigma \in \{$cl-stb$_{cf}$, cl-stb$_{adm}\}$.*

6.1.4 Symmetric irreflexive CAFs

Each symmetric irreflexive AF is coherent [32], i.e., $naive(F) = prf(F) = stb(F) = sem(F) = stg(F)$ for every symmetric irreflexive AF F. An immediate consequence is that the Nonemptiness-problem becomes trivial for all considered semantics; moreover, all range-based semantics that we consider in this paper collapse in this case.

Lemma 15. *For each symmetric irreflexive CAF CF,*

$$\begin{aligned} stb_c(CF) = {} & cl\text{-}stb_{cf}(CF) = cl\text{-}stb_{adm}(CF) \\ = {} & cl\text{-}sem(CF) = sem_c(CF) = cl\text{-}stg(CF) = stg_c(CF). \end{aligned}$$

Proof. First observe that $stb_c(CF) \neq \emptyset$ using $prf(F) = stb(F)$ for F being the underlying AF of CF. We thus obtain $cl\text{-}stb_{cf}(CF) \neq \emptyset$ and $cl\text{-}stb_{adm}(CF) \neq \emptyset$. Consequently, $cl\text{-}stb_{cf}(CF) = cl\text{-}stg(CF)$ and $cl\text{-}stb_{adm}(CF) = cl\text{-}sem(CF)$. It remains to show that $stb_c(CF) = cl\text{-}stb_{adm}(CF)$. Assume that there is $S \in cl\text{-}stb_{adm}(CF)$ that is not i-stable. Let E be a $cl\text{-}stb_{adm}$-realization of S. Since E is not stable in F, there is an argument $a \in A$ that is not attacked by E. We have $claim(a) \in S$ (otherwise, S is not cl-adm-stable in CF). By symmetry we have a does not attack E, i.e., $E \cup \{a\}$ is conflict-free. Moreover, $E \cup \{a\}$ is admissible since, in symmetric CAFs, each argument defends itself. Consequently, we can add all arguments that are unattacked by E to obtain a i-stable realization of S, contradiction to our initial assumption $S \notin stb_c(CF)$. □

We thus obtain the following complexity results as an immediate consequence from Lemma 15 and [12].

Proposition 38. *For symmetric irreflexive CAFs,* $Cred_\sigma^\Delta$, $\Delta \in \{CAF, wf\}$, $Skept_\sigma^{wf}$, *and* Ver_σ^{wf} *is in* P*;* $Skept_\sigma^{CAF}$ *is* coNP*-complete; and* Ver_σ^{CAF} *is* NP*-complete for* $\sigma \in \{cl\text{-}stb_{cf}, cl\text{-}stb_{adm}, cl\text{-}sem, sem_c, cl\text{-}stg, stg_c\}$.

We note that inherited and claim-level preferred (naive) semantics do not necessarily coincide: As a counter-example consider the CAF $CF = (\{a_1, a_2, b\}, \{(b, a_1), (a_1, b)\}, claim)$ with $claim(a_i) = a$, $claim(b) = b$, then $prf_c(CF) = \{\{a\}, \{a, b\}\} \neq \{\{a, b\}\} = cl\text{-}prf(CF)$. The respective decision problems are as hard as in the general case, using the fact that $cl\text{-}naive(CF) = cl\text{-}prf(CF)$ for every symmetric CAF (cf.Lemma 14) and the observation that the corresponding reductions for symmetric CAFs are indeed irreflexive.

Proposition 39. *For symmetric irreflexive CAFs,* $Cred_\sigma^\Delta$, $\Delta \in \{CAF, wf\}$, $Skept_\sigma^{wf}$ *and* Ver_σ^{wf} *is in* P*;* $Skept_\sigma^{CAF}$ *is* Π_2^P*-complete; and* Ver_σ^{CAF} *is* DP*-complete for* $\sigma \in \{cl\text{-}naive, cl\text{-}prf\}$.

6.1.5 Bipartite CAFs

Finally, we consider bipartite CAFs. First recall that in bipartite AFs, $prf(F) = stb(F) = sem(F) = stg(F)$. We thus obtain the following result.

Lemma 16. *For each bipartite CAF CF,*

$$prf_c(CF) = stb_c(CF) = cl\text{-}stb_{adm}(CF) = cl\text{-}sem(CF) = sem_c(CF) = stg_c(CF).$$

Proof. Let $S \in cl\text{-}stb_{adm}(CF)$. Let E be a $cl\text{-}stb_{adm}$-realization of S. By monotonicity of the claim-range we can assume $E \in prf(F)$. Thus $S = claim(E) \in stb_c(CF)$. By $cl\text{-}stb_c(CF) \neq \emptyset$, we have $cl\text{-}stb_{adm}(CF) = cl\text{-}sem(CF)$. □

By the respective problems for Dung AFs [15] and by [12], we thus obtain the following results for $\sigma \in \{cl\text{-}stb_{adm}, cl\text{-}sem, sem_c, stg_c\}$.

Proposition 40. *For bipartite CAFs, for* $\Delta \in \{CAF, wf\}$*, for* $\sigma \in \{cl\text{-}stb_{adm}, cl\text{-}sem, sem_c, stg_c\}$, $Cred_\sigma^\Delta$ *is in* P *and* $Skept_\sigma^\Delta$ *is* coNP*-complete; moreover,* Ver_σ^{CAF} *is* NP*-complete and* Ver_σ^{wf} *is in* P.

Observe that $cl\text{-}stb_{cf}(CF) \neq cl\text{-}stb_{adm}(CF)$ (as a counter-example, consider the CAF $CF = (\{a_1, a_2, b\}, \{(a_1, b)\}, claim)$ with $claim(a_i) = a$, $claim(b) = b$).

By Lemma 16 we obtain that $stb_c(CF) \neq \emptyset$ (using $prf_c(CF) = stb_c(CF)$ and $prf_c(CF) \neq \emptyset$ for all CAFs CF). Since each stable extension is non-empty, we obtain that each preferred extension is non-empty. Also, bipartite CAFs do not contain self-attacking arguments. Thus, for $\Delta \in \{CAF, wf\}$, NE_σ^Δ is a trivial yes-instance for all considered semantics σ.

By $stb_c(CF) \neq \emptyset$, we have $\textit{cl-stb}_{cf}(CF) = \textit{cl-stg}(CF)$. For well-formed CAFs, we have $stb_c(CF) = \textit{cl-stb}_{cf}(CF) = \textit{cl-stg}(CF)$ and $\textit{cl-prf}(CF) = prf_c(CF)$ for each well-formed CAF CF. By known results for i-stable semantics we thus obtain the following results for the respective reasoning problems; for cl-naive semantics, we obtain coNP-hardness for skeptical acceptance by a reduction from monotone 3-SAT via an appropriate adaption of the reduction from the proof of Proposition 14.

Proposition 41. *For bipartite CAFs, for* $\sigma \in \{\textit{cl-stb}_{cf}, \textit{cl-stg}, \textit{cl-prf}, \textit{cl-naive}\}$, $\textit{Cred}^{wf}_\sigma$ *and* $\textit{Ver}^{wf}_\sigma$ *is in* P*, and* $\textit{Skept}^{wf}_\sigma$ *is* coNP*-complete.*

Proof. coNP-hardness for skeptical acceptance of cl-naive semantics is proven analogous to [12, Proposition 17]. □

Turning now to cl-naive and cl-preferred semantics for general bipartite CAFs, we observe that that (1) the Reduction 1 is bipartite (this yields the hardness-results for cl-naive semantics) and (2) the constructed CAF in Reduction 1 satisfies $\textit{cl-naive}(CF) = \textit{cl-prf}(CF)$. We furthermore show DP-hardness by a reduction from SAT-UNSAT.

Proposition 42. *For bipartite CAFs,* $\textit{Cred}^{CAF}_\sigma$ *is in* P*,* $\textit{Skept}^{CAF}_\sigma$ *is* Π^{P}_2*-complete, and* $\textit{Ver}^{CAF}_\sigma$ *is* DP*-complete for* $\sigma \in \{\textit{cl-prf}, \textit{cl-naive}\}$.

Proof. To show DP-hardness of $\textit{Ver}^{CAF}_\sigma$, we present the following reduction from SAT-UNSAT. Consider an instance (φ_1, φ_2) where φ_i is a 3-CNF given by clauses C_i (we enumerate the clauses as follows: $C_1 = \{c_1, \ldots, c_m\}$, $C_2 = \{c_{m+1}, \ldots, c_n\}$) over atoms in X_i. We use the following construction for both φ_1 and φ_2, i.e., we construct two CAFs CF_1, CF_2 as follows: Given a CNF ψ with clauses $C = \{c_1, \ldots, c_n\}$ over atoms in X. We denote $\neg x$ by $\bar{x}$. Let $V = \{v_i \mid v \in c_i, i \leq n\}$. We construct $CF = (A, R, \textit{claim})$ with

$$\begin{aligned} A &= V \cup C \cup \{\psi\} \\ R &= \{(x_i, \bar{x}_j), (\bar{x}_j, x_i) \mid x_i, \bar{x}_j \in V\} \cup \{(c_i, \psi), (\psi, c_i) \mid i \leq n\} \end{aligned}$$

with claims $\textit{claim}(v_i) = \textit{claim}(c_i) = i$, $\textit{claim}(\psi) = \psi$. For CF, we have (1) $\textit{cl-prf}(CF) = \textit{cl-naive}(CF)$, (2) ψ is satisfiable iff $\{1, \ldots, n, \psi\}$ is cl-preferred, and (3) ψ is unsatisfiable iff $\{1, \ldots, n\}$ is cl-preferred. We obtain φ_1 is satisfiable and φ_2 is unsatisfiable iff $\{1, \ldots, n, \varphi_1\}$ is cl-preferred in $CF_1 \cup CF_2$. □

For cl-*cf*-stable and cl-stage semantics, we obtain the following results.

Proposition 43. *For bipartite CAFs, for* $\sigma \in \{\textit{cl-stb}_{cf}, \textit{cl-stg}\}$, $\textit{Cred}^{CAF}_\sigma$ *and* $\textit{Ver}^{CAF}_\sigma$ *is* NP*-complete, and* $\textit{Skept}^{CAF}_\sigma$ *is* coNP*-complete.*

Proof. Membership results follow from the respective problems for cl-*cf*-stable semantics for general CAFs (cf. Table 4).

For hardness, we first observe that $stb_c(CF) = \textit{cl-stb}_{cf}(CF)$ in the proof of [12, Proposition 2] which yields NP-completeness of $\textit{Ver}^{CAF}_\sigma$; moreover, we can adapt the proof from [12, Proposition 17] to show coNP-hardness for $\textit{Skept}^{CAF}_\sigma$.

To show NP-hardness of $\textit{Cred}^{CAF}_\sigma$, we present a reduction from SAT: Given a CNF φ with clauses $C = \{c_1, \ldots, c_n\}$ over atoms in X. We denote $\neg x$ by $\bar{x}$. Let $V = \{v_i \mid v \in c_i, i \leq n\}$. We construct $CF = (A, R, \textit{claim})$ with

$$\begin{aligned} A &= V \cup C \cup \{\varphi\} \\ R &= \{(x_i, \bar{x}_j), (\bar{x}_j, x_i) \mid x_i, \bar{x}_j \in V\} \cup \{(c_i, \varphi) \mid i \leq n\} \end{aligned}$$

with $claim(v_i) = i$, $claim(c_i) = i$, and $claim(\varphi) = \varphi$. We show that φ is credulously acceptable iff φ is satisfiable.

First assume φ is satisfiable. Then there is a model M that satisfies each clause c_i. Let $E = \{x_i \in V \mid x \in M\} \cup \{\bar{x}_i \in V \mid x \notin M\} \cup \{\varphi\}$. Clearly, E is conflict-free, moreover, $claim(E) = \{1, \dots, n, \varphi\} = claim(A)$ thus we have found a cl-*cf*-stable extension containing φ.

In case φ is credulously acceptable, let S denote the cl-*cf*-stable extension and E its realization in the underlying AF. First, $C \nsubseteq E$ because φ is the unique argument with claim φ. Thus, for each $c_i \in C$, there is an argument x_i or $\bar{x}_i$ that is contained in E. Consider the set $M = \{x \in X \mid \exists j : x_j \in E\}$. It can be shown that M is indeed a model of φ. □

This concludes our complexity analysis for graph classes. Table 8 and Table 9 summarize our results for CAFs respectively well-formed CAFs when restricted to the considered graph classes. Recall that the non-emptiness is trivial for acyclic, symmetric & irreflexive as well as for bipartite CAFs. For the remaining graph classes, i.e., for noeven and symmetric CAFs, the non-emptiness problem is tractable for all semantics except for cl-stable variants.

When comparing the different graph classes, it is not surprising that acyclic CAFs are computationally-wise the best choice for computing standard reasoning tasks; here, all considered reasoning problems for all except naive semantics are tractable. When restricted to well-formed CAFs, symmetric & irreflexive CAFs are even easier to handle; here, all considered problems are in P. In symmetric CAFs, on the other hand, almost all semantics retain their full complexity, the only exception is preferred semantics for which verification drops one level in the polynomial hierarchy (as it corresponds to verifying naive extensions in symmetric CAFs). Noeven CAFs turn out to be beneficial for computing admissible-based semantics – in this graph class, all admissible-based semantics are tractable. In symmetric & irreflexive CAFs, credulous reasoning becomes tractable; also, both variants of semi-stable and stage semantics drop one level in the polynomial hierarchy. We observe a similar behavior for bipartite CAFs, here, credulous reasoning for cl-*cf*-semantics and cl-stage semantics remains harder. Considering bipartite well-formed CAFs, skeptical reasoning for all considered semantics is coNP-complete while credulous reasoning and verification become tractable.

6.2 Fixed-parameter tractability w.r.t. the number of claims

Here we investigate well-formed CAFs with a relatively small number of claims when compared to the number of arguments. For the standard inherited semantics it has been shown that reasoning in well-formed CAFs is fixed-parameter tractable w.r.t. the number of claims used in the CAF [12]. That is, the complexity of reasoning mainly depends on the number of claims rather than the total size of the CAF. In following we

(a) extend these results to inherited semi-stable and stage semantics as well as claim-based semantics and

(b) complement existing negative results in that direction for general CAFs.

First recall that on well-formed CAFs we have that $\textit{cl-prf} = \textit{prf}_c$ and $\textit{cl-stb}_{cf} = \textit{cl-stb}_{adm} = \textit{stb}$. It thus suffices to consider *cl-naive*, $\textit{stg}_c$, *cl-stg*, $\textit{sem}_c$, *cl-sem* semantics in this section.

First, we consider the non-emptiness problem NE_σ^{wf}. The problem is already tractable for most of the considered semantics and it thus only remains to consider $\sigma \in \{\textit{sem}_c, \textit{cl-sem}\}$.

Proposition 44. *For $\sigma \in \{\textit{sem}_c, \textit{cl-sem}, \textit{cl-prf}\}$, the NE_σ^{wf} problem can be solved in time $\mathcal{O}(2^k \cdot poly(n))$ (where $poly(\cdot)$ is a fixed polynomial and n the size of the instance) for a well-formed $CAF = (A, R, claim)$ with $|claim(A)| \leq k$.*

Table 8: Complexity of CAFs with special graph structure.

graph class	task	sem_c	stg_c	$cl\text{-}naive$	$cl\text{-}prf$	$cl\text{-}stb_{cf}$	$cl\text{-}stb_{adm}$	$cl\text{-}sem$	$cl\text{-}stg$
acyclic	$Cred_\sigma^{CAF}$	in P	in P	in P	in P	in P	in P	in P	in P
	$Skept_\sigma^{CAF}$	in P	in P	Π_2^P-c	in P	in P	in P	in P	in P
	Ver_σ^{CAF}	in P	in P	DP-c	in P	in P	in P	in P	in P
	NE_σ^{CAF}	trivial for all considered semantics							
noeven	$Cred_\sigma^{CAF}$	in P	Σ_2^P-c	in P	in P	NP-c	in P	in P	Σ_2^P-c
	$Skept_\sigma^{CAF}$	in P	Π_2^P-c	Π_2^P-c	in P	coNP-c	in P	in P	Π_2^P-c
	Ver_σ^{CAF}	in P	Σ_2^P-c	DP-c	in P	NP-c	in P	in P	Σ_2^P-c
	NE_σ^{CAF}	in P	in P	in P	in P	NP-c	in P	in P	in P
symmetric & irreflexive	$Cred_\sigma^{CAF}$	in P	in P	in P	in P	in P	in P	in P	in P
	$Skept_\sigma^{CAF}$	coNP-c	coNP-c	Π_2^P-c	Π_2^P-c	coNP-c	coNP-c	coNP-c	coNP-c
	Ver_σ^{CAF}	NP-c	NP-c	DP-c	DP-c	NP-c	NP-c	NP-c	NP-c
	NE_σ^{CAF}	trivial for all considered semantics							
symmetric	$Cred_\sigma^{CAF}$	Σ_2^P-c	Σ_2^P-c	in P	in P	NP-c	NP-c	Σ_2^P-c	Σ_2^P-c
	$Skept_\sigma^{CAF}$	Π_2^P-c	Π_2^P-c	Π_2^P-c	Π_2^P-c	coNP-c	coNP-c	Π_2^P-c	Π_2^P-c
	Ver_σ^{CAF}	Σ_2^P-c	Σ_2^P-c	DP-c	DP-c	NP-c	NP-c	Σ_2^P-c	Σ_2^P-c
	NE_σ^{CAF}	in P	in P	in P	in P	NP-c	NP-c	in P	in P
bipartite	$Cred_\sigma^{CAF}$	in P	in P	in P	in P	NP-c	in P	in P	NP-c
	$Skept_\sigma^{CAF}$	coNP-c	coNP-c	Π_2^P-c	Π_2^P-c	coNP-c	coNP-c	coNP-c	coNP-c
	Ver_σ^{CAF}	NP-c	NP-c	DP-c	DP-c	NP-c	NP-c	NP-c	NP-c
	NE_σ^{CAF}	trivial for all considered semantics							

Table 9: Complexity of well-formed CAFs with special graph structure.

graph class	task	sem_c	stg_c	*cl-naive*	*cl-prf*	*cl-stb*$_{cf}$	*cl-stb*$_{adm}$	*cl-sem*	*cl-stg*
acyclic	$Cred_\sigma^{wf}$	in P	in P	in P	in P	in P	in P	in P	in P
	$Skept_\sigma^{wf}$	in P	in P	coNP-c	in P	in P	in P	in P	in P
	Ver_σ^{wf}	in P	in P	in P	in P	in P	in P	in P	in P
	NE_σ^{CAF}	trivial for all considered semantics							
noeven	$Cred_\sigma^{wf}$	in P	Σ_2^P-c	in P	in P	in P	in P	in P	Σ_2^P-c
	$Skept_\sigma^{wf}$	in P	Π_2^P-c	coNP-c	in P	in P	in P	in P	Π_2^P-c
	Ver_σ^{wf}	in P	coNP-c	in P	in P	in P	in P	in P	coNP-c
	NE_σ^{CAF}	in P	in P	in P	in P	in P	in P	in P	in P
symmetric & irreflexive	$Cred_\sigma^{wf}$	in P	in P	in P	in P	in P	in P	in P	in P
	$Skept_\sigma^{wf}$	in P	in P	in P	in P	in P	in P	in P	in P
	Ver_σ^{wf}	in P	in P	in P	in P	in P	in P	in P	in P
	NE_σ^{CAF}	trivial for all considered semantics							
symmetric	$Cred_\sigma^{wf}$	Σ_2^P-c	Σ_2^P-c	in P	in P	NP-c	NP-c	Σ_2^P-c	Σ_2^P-c
	$Skept_\sigma^{wf}$	Π_2^P-c	Π_2^P-c	in P	in P	coNP-c	coNP-c	Π_2^P-c	Π_2^P-c
	Ver_σ^{wf}	coNP-c	coNP-c	in P	in P	in P	in P	coNP-c	coNP-c
	NE_σ^{CAF}	in P	in P	in P	in P	NP-c	NP-c	in P	in P
bipartite	$Cred_\sigma^{wf}$	in P	in P	in P	in P	in P	in P	in P	in P
	$Skept_\sigma^{wf}$	coNP-c	coNP-c	coNP-c	coNP-c	coNP-c	coNP-c	coNP-c	coNP-c
	Ver_σ^{wf}	in P	in P	in P	in P	in P	in P	in P	in P
	NE_σ^{CAF}	trivial for all considered semantics							

Table 10: Parameterized complexity of well-formed CAFs $CAF = (A, R, claim)$ with respect to $k = |claim(A)|$ (FPT denotes the class of fixed-parameter tractable problems).

task	sem_c	stg_c	$cl\text{-}naive$	$cl\text{-}prf$	$cl\text{-}stb_{cf}$	$cl\text{-}stb_{adm}$	$cl\text{-}sem$	$cl\text{-}stg$
$Cred^{wf}_\sigma$	in FPT	in FPT	in P	in FPT	in FPT	in FPT	in FPT	in FPT
$Skept^{wf}_\sigma$	in FPT	in FPT	in FPT	in FPT	in FPT	in FPT	in FPT	in FPT
Ver^{wf}_σ	in FPT	in FPT	in P	in FPT	in P	in P	in FPT	in FPT
NE^{CAF}_σ	in FPT	in P	in P	in FPT	in FPT	in FPT	in FPT	in P

Proof. We iterate over all sets $C \subseteq claim(A)$ and compute the corresponding candidates for an admissible set $E \subseteq A$ with $claim(E) = C$. If one of these sets is indeed admissible we return yes otherwise false. For each C this procedures is in P (cf. Lemma 7). □

We next present an enumeration algorithm for the extensions to show the upper bounds for the credulous and skeptical reasoning tasks as well as the verification problem.

Proposition 45. *For $\sigma \in \{cl\text{-}naive, stg_c, cl\text{-}stg, sem_c, cl\text{-}sem\}$, the $Cred^{wf}_\sigma$, $Skept^{wf}_\sigma$ and Ver^{wf}_σ problems can be solved in time $\mathcal{O}(4^k \cdot poly(n))$ (where $poly(\cdot)$ is a fixed polynomial and n the size of the instance) for a well-formed $CAF = (A, R, claim)$ with $|claim(A)| \leq k$.*

Proof. We iterate over all sets $C \subseteq claim(A)$ and compute the corresponding maximal conflict-free (resp. admissible) set $E \subseteq A$ in P (cf. Lemma 7) and filter out sets C that do not have a corresponding conflict-free (resp. admissible) set. We end up with at most 2^k many sets. Next, depending on the semantics σ we proceed as follows:

- For *cl-naive* we compare the remaining sets C pairwise and filter out sets that are not $\subseteq$-maximal.
- For stg_c and sem_c we compute the range for the extensions by adding all attacked arguments to E. Finally, we eliminate all pairs for which the range is not $\subseteq$ - maximal.
- For *cl-stg* and *cl-sem* we compute the claim-range for the extensions by adding all defeated claims to C. Finally, we eliminate all pairs for which the claim-range is not $\subseteq$- maximal

In all three cases we end up with the set of extensions and can now easily decide the credulous and skeptical acceptance of arguments as well as the validity of a given extension. □

These fixed-parameter tractability results are summarized in Table 10. We next show that for general CAFs and $\sigma \in \{sem_c, stg_c\}$ these problems are not fixed-parameter tractable w.r.t. number of claims but maintain their full complexity even when there are only two claims.

Proposition 46. *For $\sigma \in \{sem_c, stg_c\}$,*

- *$Cred^{CAF}_\sigma$, $Skept^{CAF}_\sigma$, Ver^{CAF}_σ maintain their full complexity even for CAFs with only two claims, and*
- *$NE^{CAF}_{sem_c}$ maintains its full complexity even for CAFs with only one claim.*

Proof. First consider the following translation for a given $CAF = (A, R, claim)$ with an arbitrary number of claims and a given claim c. Construct $CAF' = (A, R, claim')$ with $claim'(a) = c$ iff $claim(a) = c$ and $claim'(a) = d$ otherwise. Then claim c is credulously (resp. skeptically) accepted in CAF iff c is credulously (resp. skeptically) accepted in CAF'. We obtain that $Cred_\sigma^{CAF}$, $Skept_\sigma^{CAF}$ maintain their full complexity.

The lower bound for Ver_σ^{CAF} can be obtained similar as in the proof of Proposition 31 via a reduction from the Σ_2^{P}-complete $Cred_\sigma$ problem for Dung AFs. To decide the problem for an argument b in a Dung $AF = (A, R)$, construct a $CAF = (A' = A \cup \{x\}, R, claim)$ with a new argument $x \notin A$ and $claim(b) = c_1$ and $claim(a) = c_2$ for all $a \in A' \setminus \{b\}$. Then, argument b is credulously accepted in AF with regard to σ iff $\{c_1, c_2\}$ is a i-σ extension of CAF.

$NE_{sem_c}^{CAF}$: For a given $CAF = (A, R, claim)$ with an arbitrary number of claims, create $CAF' = (A, R, claim')$ with $claim'(a) = c$ for all $a \in A$. Then $NE_{sem}^{CAF}(CAF) = NE_{sem}^{CAF}(CAF')$. □

That is, for all consider inherited semantics, the problems retain their full complexity for general CAFs with only two claims. The picture for claim-based semantics is a more subtle. For instance consider *cl-prf* (respectively *cl-naive*) with just two claims $\{c_1, c_2\}$. In order to test whether c_1 is skeptically accepted it is sufficient to test whether $\emptyset$ and $\{c_2\}$ are not *cl-prf* which is in DP. That is, a small number of claims can lower the complexity of claim-based semantics. While a full investigation of this matter is beyond the scope of this paper, we observe that claim-based semantics remain NP/coNP-hard.

Proposition 47. *For CAFs with only two claims,*

- Ver_σ^{CAF} *is* NP*-hard for* $\sigma \in \{cl\text{-}stb_{cf}, cl\text{-}stb_{adm}, cl\text{-}prf\}$,
- Ver_σ^{CAF} *is* coNP*-hard for* $\sigma \in \{cl\text{-}stg, cl\text{-}sem\}$, *and*
- NE_σ^{CAF} *is* NP*-complete* $\sigma \in \{cl\text{-}stb_{cf}, cl\text{-}stb_{adm}, cl\text{-}prf, cl\text{-}stg, cl\text{-}sem\}$.

That is, for all semantics, except *cl-naive*, the parametrized approach discussed here does not lead to tractability results. Finally let us consider the case of *cl-naive*. $Ver_{cl\text{-}naive}^{CAF}$ for CAFs with only two claims can be solved in polynomial time by considering all pairs of arguments where the first argument has claim 1 and the second argument has claim 2 and check whether one of those pairs is conflict-free. Indeed this can be generalized to an $O(n^k \cdot poly(n))$ algorithm for k claims. However, this algorithm does not fall in the class of FPT but a class of higher complexity, i.e., the class XP which contains the parameterized problems with runtime $O(n^f(k))$ for some computable function f.

7 Discussion

In this work we studied the computational complexity of the different semantics for claim-augmented argumentation frameworks. That is, we complemented existing complexity results for inherited semantics [12] and provided a full complexity analysis of claim-level semantics. We want to highlight three observations here: (a) for both approaches the verification problem is harder than in the AF setting, which is in particular relevant when it comes to the enumeration of extensions; (b) however, when restricted to well-formed CAFs the complexity of verification drops to the complexity of AFs; and (c) the complexity of inherited and claim-level semantics differs for naive and preferred semantics.

Moreover, given the high complexity of the considered semantics we investigated tractable fragments in terms of certain graph classes (that are known to be tractable when neglecting claims) as well as a parameterized algorithm for enumerating extensions in well-formed CAFs. The full complexity classification of the semantics together with the first tractable fragments paves the way for complexity-adequate reduction-based implementations [33, 34, 35] of the considered semantics which is an emerging topic for future work.

Besides studying the standard reasoning tasks we also settled the complexity of the concurrence problem, i.e., deciding whether two variants of a semantics coincide on a CAF. The concurrence problem is in the tradition of the well-known coherence problem [23], which (a) for AFs is Π_2^P-complete; (b) remains Π_2^P-complete for inherited semantics [12]; and (c) also for claim-based semantics, despite the complexity increase for reasoning problems, remains Π_2^P-complete (Proposition 22). However, the complexity for the novel concurrence problem turns out to be surprisingly hard, ranging up to the third level of the polynomial hierarchy.

Concerning future work we identify the following directions. In this work we considered two different families of claim-based argumentation semantics that both followed the CAF approach of using extensions of arguments, map them to extensions of claims and then reason about the acceptance of claims. This a common approach in structured argumentation and there are more ways of lifting argument semantics to the claim-level, as recently discussed in [7]. Investigating the computational properties of these approaches is a promising direction for future research. Moreover, given the complexity of the fundamental problems for the semantics under our considerations one can reach for more advanced computational tasks, e.g., dealing with incomplete information on the arguments and attacks [36, 37], the problem of counting the number of extensions [38, 39], or enforcing the acceptance of a statement or an extension [40, 41].

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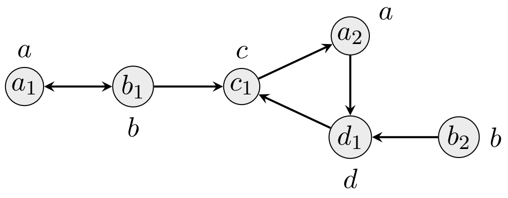
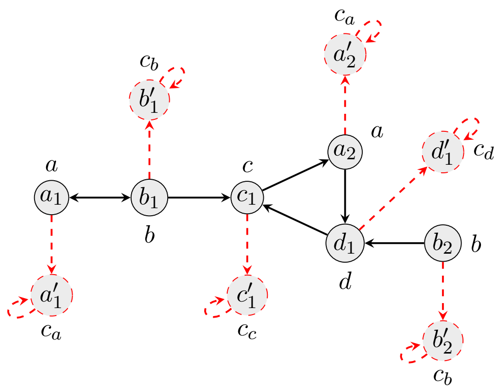
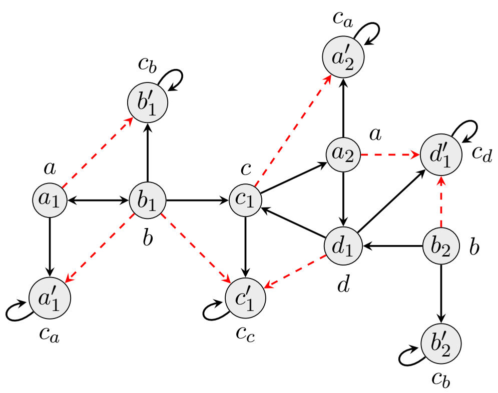


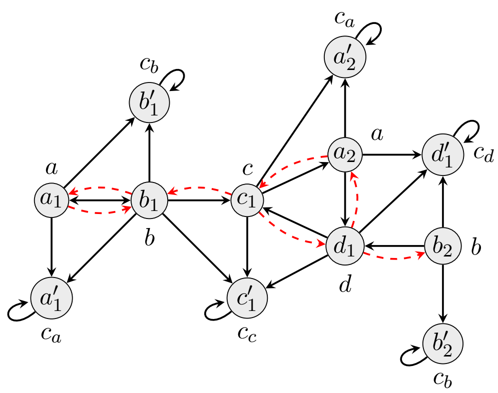
The original CAF from Example 1.



The resulting CAF after applying Tr_1 from Reduction 5 to the CAF from Example 1.



The resulting CAF after applying Tr_2 from Reduction 5 to the CAF from Example 1.



The resulting CAF after applying Tr_3 from Reduction 5 to the CAF from Example 1. Note, that the two attacks between a_1 and b_1 introduced are redundant, as the original CAF already contained those attacks, yet have been included in this figure.

Figure 6: The translations of Reduction 5 applied to the CAF from Example 1. Highlighted in red are the changes with respect to the original CAF or the previous translation.

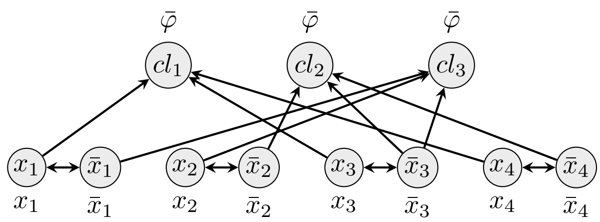


Figure 7: A CAF illustrating the reduction in the proof of Proposition 14 for the formula φ with clauses $\{\{x_1, x_3, x_4\}, \{\bar{x}_3, \bar{x}_4, \bar{x}_2)\}, \{\bar{x}_1, \bar{x}_3, x_2\}\}$.

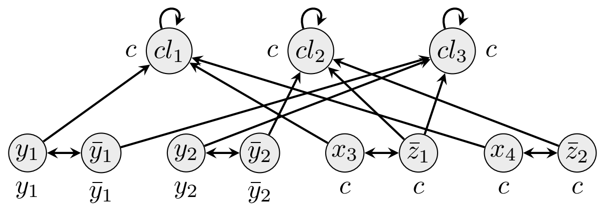


Figure 8: A CAF illustrating Reduction 6 for the formula $\Psi = \forall Y \exists Z \varphi(Y, Z)$ where $\varphi(Y, Z)$ is given by the clauses $\{\{y_1, z_1, z_2\}, \{\bar{z}_1, \bar{z}_2, \bar{y}_2)\}, \{\bar{y}_1, \bar{z}_1, y_2\}\}$.

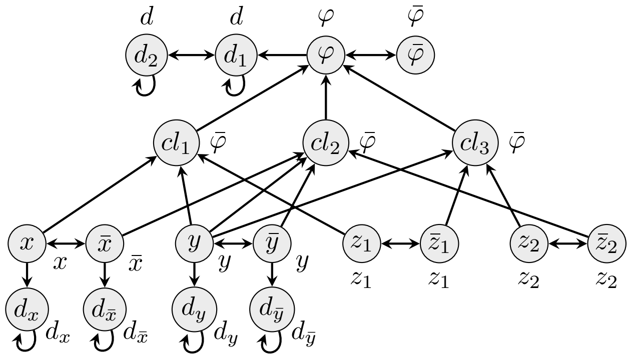


Figure 9: Reduction 7 for the formula $\exists X \forall Y \exists Z \varphi(X, Y, Z)$ with clauses $\{\{z_1, x, y\}, \{\neg x, \neg y, \neg z_2, y\}, \{\neg z_1, z_2, y\}\}$.

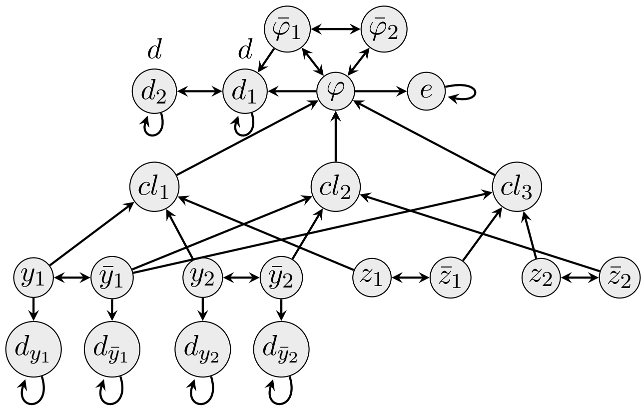


Figure 10: Reduction 8 for the formula $\forall Y \exists Z \varphi(Y, Z)$ where $\varphi(Y, Z)$ is given by the clauses $\{\{z_1, y_1, y_2\}, \{\bar{y}_1, \bar{y}_2, \bar{z}_2)\}, \{\bar{z}_1, \bar{y}_1, z_2\}\}$. Since $claim(a) = a$ for all arguments $a \in A \setminus \{d_1, d_2\}$, we omit all claims that coincide with the arguments name.

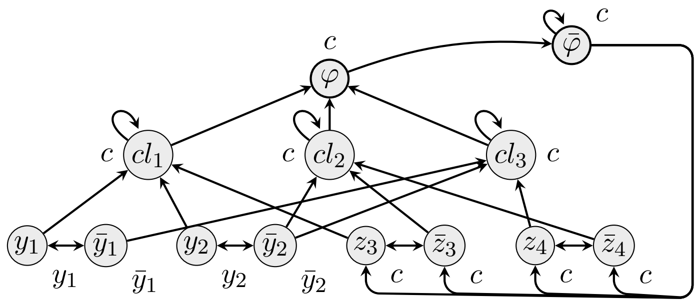


Figure 11: CAF from the proof of Proposition 21 for the QBF $\forall\{y_1, y_2\}\exists\{z_3, z_4\}$: $\{\{y_1, y_2, z_3\}, \{\bar{y}_2, \bar{z}_3, \bar{z}_4)\}, \{\bar{y}_1, \bar{y}_2, z_4\}\}$.

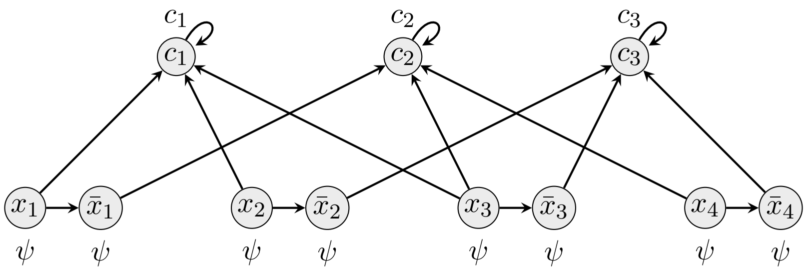


Figure 12: Reduction 9 for the formula φ given by the clauses $\{\{x_1, x_2, x_3\}, \{\bar{x}_1, x_3, x_4\}, \{\bar{x}_2, \bar{x}_3, \bar{x}_4\}\}$

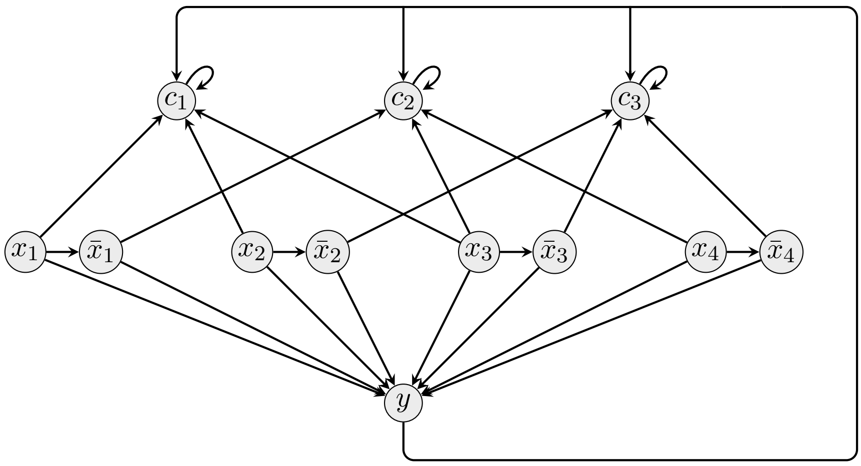


Figure 13: Reduction 10 for the formula φ given by the clauses $\{\{x_1, x_2, x_3\}, \{\bar{x}_1, x_3, x_4\}, \{\bar{x}_2, \bar{x}_3, \bar{x}_4\}\}$

A Translations between semantics (Proof of Lemma 6)

Lemma 6. *For a CAF $CF = (A, R, claim)$,*

$$prf_c(CF) = prf_c(Tr_1(CF)) = cl\text{-}sem(Tr_1(CF)),$$
$$stb_c(CF) = stb_c(Tr_2(CF)) = cl\text{-}stb_\tau(Tr_2(CF)) \text{ for } \tau \in \{adm, cf\},$$
$$stg_c(CF) = stg_c(Tr_3(CF)) = cl\text{-}stg(Tr_3(CF)).$$

The statement is proven in the following Lemmata 17, 18, and 19.

Lemma 17. *For a CAF $CF = (A, R, claim)$, $prf_c(CF) = prf_c(Tr_1(CF)) = cl\text{-}sem(Tr_1(CF))$.*

Proof. Let $Tr_1(CF) = CF' = (A', R', claim')$. The proof proceeds in three steps:

(i) We first show that $C \in cf_c(CF)$ if and only if $C \in cf_c(CF')$ and further that $prf_c(CF) = prf_c(CF')$.
$\Rightarrow$: Let E be a cf_c-realization of C in (A, R). As $E \subseteq A$, it cannot contain any a'. Thus, $E \in cf((A', R'))$, as all additional attacks contain at least one argument a', which are not contained in E and therefore $C \in cf_c(CF')$.
$\Leftarrow$: Let E be a cf_c-realization of C in (A', R'). As all arguments a' are self-attacking, $E \cap A' = \emptyset$. Therefore, as $R \subseteq R'$, $E \in cf((A, R))$ and thus $C \in cf_c(CF)$.

Moreover, also $E \in adm((A, R))$ if and only if $E \in adm((A', R'))$, as $E \cap A' = \emptyset$. Now, as preferred extensions are subset maximal admissible sets, we further obtain that $E \in prf((A, R))$ if and only if $E \in prf((A', R'))$ and thus, $prf_c(CF) = prf_c(CF')$.

(ii) Next, to show that $prf_c(CF') \subseteq cl\text{-}sem(CF')$, let $C \in prf_c(CF')$ and E be a prf_c-realization of C in (A', R'). Furthermore, towards a contradiction, let $F \in adm((A', R'))$ and $C \cup \nu_{CF'}(E) \subset claim'(F) \cup \nu_{CF'}(F)$. As $E \in prf((A', R'))$, there must be some $a \in E \setminus F$. Furthermore, as all arguments $b' \in A' \setminus A$ are self-attacking, it must hold that $a \in A$ and thus, by the construction of Tr_1, there must be some argument a' such that a is the only argument attacking a' and a' is the only argument with claim $claim'(a')$. Therefore, $claim'(a') \in \nu_{CF'}(E)$ but $claim(a') \notin claim'(F) \cup \nu_{CF'}(F)$, contradicting that $C \cup \nu_{CF'}(E) \subset claim'(F) \cup \nu_{CF'}(F)$. Thus, such a set F cannot exist and therefore, $prf_c(CF') \subseteq cl\text{-}sem(CF')$.

(iii) Finally, to show that $cl\text{-}sem(CF') \subseteq prf_c(CF')$, let $C \in cl\text{-}sem(CF')$ and $E \subseteq A'$ be a admissible set witnessing C. Towards a contradiction, let $F \subseteq prf((A', R'))$ such that $E \subset F$. Then, $C \cup \nu_{CF'}(E) \subseteq claim'(F) \cup \nu_{CF'}(F)$. Furthermore, as $E \subset F$, there must be some $a \in F \setminus E$ and thus some $a' \in A'$ attacked by a. As, by the construction of Tr_1, a' is the only argument with claim $claim'(a')$ and is only attacked by a (except for itself), $claim'(a') \in claim'(F) \cup \nu_{CF'}(F)$ and $claim'(a') \notin C \cup \nu_{CF'}(E)$ and thus $C \cup \nu_{CF'}(E) \subset claim'(F) \cup \nu_{CF'}(F)$, contradicting that $C \in cl\text{-}sem(CF')$. Thus, such a set F cannot exist and therefore, $cl\text{-}sem(CF') \subseteq prf_c(CF')$. □

Lemma 18. *For a CAF $CF = (A, R, claim)$, $stb_c(CF) = stb_c(Tr_2(CF)) = cl\text{-}stb_\tau(Tr_2(CF))$ for $\tau \in \{adm, cf\}$.*

Proof. Let $Tr_2(CF) = CF' = (A', R', claim')$. Since $stb_c(CF) \subseteq cl\text{-}stb_{adm}(CF) \subseteq cl\text{-}stb_{cf}(CF)$ holds for any CAF CF, it suffices to show that (i) $stb_c(CF) \subseteq stb_c(CF')$ and (ii) $cl\text{-}stb_{cf}(CF') \subseteq stb_c(CF)$.

First observe that (a) for every set of arguments $E \subseteq A$, E attacks the argument a' in CF' iff $a \in E \cup E^+_{(A,R)}$. Indeed, E attacks an argument a' iff either $a \in E$ or if there is $b \in E$ such that $(b, a) \in R$.

(i) Let $S \in stb_c(CF)$ and consider a stb_c-realization $E \subseteq A$. We show that E is stable in CF': First notice that E is conflict-free since we introduced no attacks between existing arguments in CF'. Moreover, E attacks every argument $a \in A' \setminus E$: Clearly, E attacks every argument $a \in A \setminus E$; moreover, E attacks every $a' \in \{a' \mid a \in A\}$ by (a) since $E \cup E^+_{(A,R)} = A$.

(ii) Let $S \in \mathit{cl\text{-}stb}_{cf}(CF')$, then there is a set $E \in A'$ such that $E \in \mathit{cf}((A', R'))$ and $\mathit{claim}(E) \cup \nu_{CF'}(E) = \mathit{claim}(A')$. We show that $E \in \mathit{stb}((A, R))$. First observe that $E \subseteq A$ since each argument $a' \in \{a' \mid a \in A\}$ is self-attacking; moreover, E is conflict-free in (A, R). We show that E attacks every argument $a \in A \setminus E$: We have $\{c_a \mid a \in A\} \subseteq \nu_{CF'}(E)$ since $\mathit{claim}(E) \cup \nu_{CF'}(E) = \mathit{claim}(A')$. Thus E attacks each argument a' in CF'. We conclude by (a) that $a \in E \cup E^+_{(A,R)}$ for every argument $a \in A$. We have shown that $E \in \mathit{stb}((A, R))$ and, consequently, $S \in \mathit{stb}_c(CF)$. □

Lemma 19. *For a CAF $CF = (A, R, \mathit{claim})$,*
$\mathit{stg}_c(CF) = \mathit{stg}_c(Tr_3(CF)) = \mathit{cl\text{-}stg}(Tr_3(CF))$.

Proof. Let $Tr_3(CF) = CF' = (A', R', \mathit{claim}')$. The proof proceeds in three steps:

(i) First, observe that $\mathit{cf}((A, R)) = \mathit{cf}((A', R'))$ as all added arguments are self-attacking and we only add attacks between arguments $\{a, b\} \subseteq A$ if there was already one in at least one direction or the attacked argument was self-attacking. Moreover, $\{\emptyset\} \in \mathit{stg}_c(CF)$ if and only if all arguments are self-attacking which is the case if and only if $\{\emptyset\} \in \mathit{cl\text{-}stg}(CF)$.

(ii) Regarding $\mathit{stg}_c(CF) = \mathit{stg}_c(CF')$: For every maximal (with regard to $\subseteq$) $E \in \mathit{cf}(A', R')$, $A \subseteq E \cup E^+_{(A',R')}$, as all arguments in A are either contained or, due to the fact that E is maximal, are attacked by E. Thus, such sets E, due to the fact that all arguments a' are self-attacking, are the only witnessing candidates for the extensions in $\mathit{stg}_c(CF)$ and $\mathit{stg}_c(CF')$. Furthermore, by construction of Tr_3, $E \cup E^+_{(A',R')} = A \cup \{a' \in A' \mid a \in E \cup E^+_{(A,R)}\}$ and thus $E \cup E^+_{(A,R)}$ will be maximal if and only if $E \cup E^+_{(A',R')}$ is maximal.

(iii) Finally, $\mathit{stg}_c(CF') = \mathit{cl\text{-}stg}(CF')$ follows by observing that the claims of all arguments in A' are unique. □

B Concurrence for stage semantics (Proof of Lemma 10)

Below we prove the correspondence of semi-stable and stage semantics for CAFs generated from Reduction 7. This lemma is the main part for proving Π^{P}_3-hardness for Con^{CAF}_{stg}.

Lemma 10. *Let $\Psi = \exists X \forall Y \exists Z \varphi(X, Y, Z)$ be an instance of $QSAT^{\exists}_3$ and let $CF = (A, R, \mathit{claim})$ be as in Reduction 7. Then*

1. *$\mathit{cl\text{-}sem}(CF) = \mathit{cl\text{-}stg}(CF)$; and*
2. *$\mathit{sem}_c(CF) = \mathit{stg}_c(CF)$.*

Proof. To prove the statements we will first show that (i) each cl-stage and each i-stage claim-set is of the form $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\}$ for some $X' \subseteq X$ and for $e \in \{\varphi, \bar{\varphi}\}$: Let $S \in \mathit{stg}_c(CF) \cup \mathit{cl\text{-}stg}(CF)$, $V = X \cup Y \cup Z$. First notice that $S \subseteq X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\}$ for some $X' \subseteq X$, for $e \in \{\varphi, \bar{\varphi}\}$: S cannot contain both $a, \bar{a}$ for $a \in X \cup \{\varphi\}$ since there is no cf_c-realization E containing both $b, \bar{b}$, for $b \in X$, nor φ, b for $b \in \{\bar{\varphi}\} \cup \mathcal{C}$. It remains to show that $X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\} \subseteq S$ for some $X' \subseteq X$, for $e \in \{\varphi, \bar{\varphi}\}$.

Let $S \in \mathit{stg}_c(CF)$ and consider a stg_c-realization E of S. E contains $V' \cup \{\bar{v} \mid v \notin V'\}$ for some $V' \subseteq V$: Assume there is $v \in V$ such that $v, \bar{v} \notin E$ and let $D = (E \setminus \{cl_i \mid (v, cl_i) \in R\}) \cup \{v\}$. D is conflict-free since $\bar{v}, d_v \notin E$ and since $cl_i \notin E$ for each clause cl_i with $(v, cl_i) \in R$. Moreover, each such cl_i is attacked by D and thus $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, contradiction to E being stage in (A, R). Moreover, E contains either φ or $\bar{\varphi}$: Towards a contradiction, assume $\varphi, \bar{\varphi} \notin E$ and let $D = E \cup \{\bar{\varphi}\}$. D is conflict-free since $\varphi \notin E$ and $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, contradiction to E being stage in (A, R).

Let $S \in \textit{cl-stg}(CF)$. We will first show that S contains either φ or $\bar{\varphi}$: Towards a contradiction, assume $\varphi, \bar{\varphi} \notin S$. As S is cl-stage, there is an cf_c-realization E of S such that $\textit{claim}(E) \cup \nu_{CF}(E)$ is maximal among conflict-free claim-sets. Let $D = E \cup \{\bar{\varphi}\}$. D is conflict-free since $\varphi \notin E$ and thus $\textit{claim}(D) \cup \nu_{CF}(D) = \textit{claim}(E) \cup \nu_{CF}(E) \cup \{\varphi, \bar{\varphi}\} \supset \textit{claim}(E) \cup \nu_{CF}(E)$, contradiction to S being cl-stage. S contains $X' \cup \{\bar{x} \mid x \notin X'\}$ and $Y \cup Z \subseteq S$: Assume there is $x \in X$ such that $x, \bar{x} \notin S$. As S is cl-stage, there is an cf_c-realization E of S such that $\textit{claim}(E) \cup \nu_{CF}(E)$ is maximal among conflict-free claim-sets. In case $\varphi \in S$, then $\varphi \in E$ and $\bar{\varphi} \notin E$, $cl_i \notin E$, $i \leq n$, since they are in conflict with φ. Then $D = E \cup \{x\}$ is conflict-free and properly extends E, thus $\textit{claim}(D) \cup \nu_{CF}(D) \supset \textit{claim}(E) \cup \nu_{CF}(E)$, contradiction to S being cl-stage. In case $\bar{\varphi} \in E$, let $D = (E \setminus \{cl_i \mid (x, cl_i) \in R\}) \cup \{x, \bar{\varphi}\}$. D is conflict-free since $\bar{x}, d_x \notin E$, $cl_i \notin E$ for each clause cl_i with $(v, cl_i) \in R$ and $\varphi \notin E$ by assumption $\bar{\varphi} \in S$. $\textit{claim}(D) = \textit{claim}(E) \cup \{x\}$ since the only arguments which have been removed from D are labelled with claim $\bar{\varphi}$ and D contains $\bar{\varphi}$; moreover, $\nu_{CF}(E) \subseteq \nu_{CF}(D)$ since φ is the only attacked argument of each cl_i and $(\bar{\varphi}, \varphi) \in R$. Consequently, $\textit{claim}(D) \cup \nu_{CF}(D) \supset \textit{claim}(E) \cup \nu_{CF}(E)$, contradiction to S being cl-stage. $Y \cup Z \subseteq S$: Assume there is $v \in Y \cup Z$ such that $v \notin S$. As S is cl-stage, there is an cf_c-realization E of S such that $\textit{claim}(E) \cup \nu_{CF}(E)$ is maximal among conflict-free claim-sets and E does not contain $v, \bar{v}$ by assumption. Analogous to above, one can extend E appropriately to derive a contradiction to S being cl-stage.

(1) Analogous to Lemma 9, one can show that $\textit{cl-stg}(CF) = \{X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{e\} \mid X' \subseteq X, e \in \{\varphi, \bar{\varphi}\}\}$.

(2) We will show (a) $\textit{stg}_c(CF) \subseteq \textit{sem}_c(CF)$; and (b) $\textit{sem}_c(CF) \subseteq \textit{stg}_c(CF)$.

To show (a), let $S \in \textit{stg}_c(CF)$. By (i), either $\varphi \in S$ or $\bar{\varphi} \in S$. In case $\varphi \in S$, we have $S = X' \cup \{\bar{x} \mid x \notin X'\} \cup Y \cup Z \cup \{\varphi\}$ for some $X' \subseteq X$, thus $S \in \textit{sem}_c(CF)$ by Lemma 9. In case $\bar{\varphi} \in S$, we consider a $\textit{stg}_c$-realization E of S. E is admissible: Each $a \in V \cup \bar{V} \cup \{\bar{\varphi}\}$ defends itself; also, $\varphi \notin E$ by (i); moreover, each $cl_i \in E$ is defended by E, otherwise there is $cl_i \in E$ which is not defended by E against some argument $a \in V \cup \bar{V}$, thus $\bar{a} \notin E$, that is, there is $v \in V$ such that $v, \bar{v} \notin E$, contradiction to (i). Thus E is semi-stable, otherwise there is some set $D \in \textit{adm}((A, R)) \subseteq \textit{cf}((A, R))$ with $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, contradiction to E being stage in (A, R).

To show (b), let $S \in \textit{sem}_c(CF)$ and consider a $\textit{sem}_c$-realization E of S. Clearly, E is conflict-free. We show that $E \in \textit{stg}((A, R))$. Towards a contradiction, assume that there is $D \in \textit{cf}((A, R))$ with $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$. Let $a \in D^{\oplus}_{(A,R)} \setminus E^{\oplus}_{(A,R)}$. By Lemma 8, either $E^{\oplus}_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_2\})$ (in case $\varphi \in E$) or $E^{\oplus}_{(A,R)} = A \setminus (\{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_1, d_2\})$ (in case $\bar{\varphi} \in E$); that is, $a \in \{d_a \mid a \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\} \cup \{d_1, d_2\}$. Also, for all $v \in V$, either $d_v \in E^{\oplus}_{(A,R)}$ or $d_{\bar{v}} \in E^{\oplus}_{(A,R)}$, otherwise $v, \bar{v} \notin E$; let $E' = E \cup \{v\}$, then $(E')^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, contradiction to E being semi-stable. In case $a = d_b$ for some $b \in (X \cup \bar{X} \cup Y \cup \bar{Y}) \setminus E\}$, we have $d_b, d_{\bar{b}} \in D^{\oplus}_{(A,R)}$ and thus $b, \bar{b} \in D$, contradiction to D being conflict-free. Moreover, $a \neq d_2$ since the only attacker d_1 of d_2 is self-attacking. Consider the case $a = d_1$, then $\varphi \in D$ since φ is the only attacker of d_1. Thus $cl_i \notin D$ for all $i \leq n$ by conflict-freeness of D; we conclude that D attacks each cl_i, $i \leq n$ since $cl_i \in E^{\oplus}_{(A,R)}$ for all $i \leq n$ and $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$. Therefore D is admissible and $D^{\oplus}_{(A,R)} \supset E^{\oplus}_{(A,R)}$, contradiction to E being semi-stable. □

C Bounding the number of claims (Proof of Proposition 47)

Proposition 47. *For CAFs with only two claims,*

- Ver_{σ}^{CAF} *is* NP*-hard for* $\sigma \in \{cl\text{-}stb_{cf}, cl\text{-}stb_{adm}, cl\text{-}prf\}$,
- Ver_{σ}^{CAF} *is* coNP*-hard for* $\sigma \in \{cl\text{-}stg, cl\text{-}sem\}$, *and*
- NE_{σ}^{CAF} *is* NP*-complete* $\sigma \in \{cl\text{-}stb_{cf}, cl\text{-}stb_{adm}, cl\text{-}prf, cl\text{-}stg, cl\text{-}sem\}$.

Proof. The hardness proofs for Ver_{σ}^{CAF} are by three variants of the standard reduction:

$\sigma \in \{cl\text{-}stb_{cf}, cl\text{-}stb_{adm}\}$: Let φ be an instance of 3-SAT, with φ given as a set of clauses $C = \{c_1, \dots, c_n\}$ over atoms in X, where negated atoms are denoted by $\bar{x}$. We construct $CAF_{\varphi} = (A, R, claim)$ with

$$\begin{aligned} A &= X \cup \bar{X} \cup C \\ R &= \{(x, \bar{x}), (\bar{x}, x) \mid x \in X\} \cup \{(l, c) \mid c \in C, l \in c\} \cup \{(c, c) \mid c \in C\} \end{aligned}$$

with $claim(x) = claim(\bar{x}) = c$ for all $x \in X$ and $claim(c_i) = d$ for all $c_i \in C$. An illustrative example of the reduction is given in Figure 14. First notice that because of the specific use of symmetric attacks and the self attacks conflict-free sets and admissible sets coincide. Thus, also $cl\text{-}stb_{cf}$ and $cl\text{-}stb_{adm}$ coincide and it suffices to consider $cl\text{-}stb_{cf}$ in the following. By construction the formula φ is satisfiable iff there is a conflict-free set that attacks all arguments $c_i \in C$ iff there is a $cl\text{-}stb_{cf}$ extension iff $\{c\}$ is a $cl\text{-}stb_{cf}$ extension. We obtain that Ver_{σ}^{CAF} is NP-hard.

$\sigma \in \{cl\text{-}prf\}$: Let φ be an instance of 3-SAT, with φ given as a set of clauses $C = \{c_1, \dots, c_n\}$ over atoms in X, where negated atoms are denoted by $\bar{x}$. We construct $CAF_{\varphi} = (A, R, claim)$ with

$$\begin{aligned} A &= X \cup \bar{X} \cup C \cup \{\varphi\} \\ R &= \{(x, \bar{x}), (\bar{x}, x) \mid x \in X\} \cup \{(l, c) \mid c \in C, l \in c\} \cup \{(c, c), (c, \varphi) \mid c \in C\} \end{aligned}$$

with $claim(x) = claim(\bar{x}) = c$ for all $x \in X \cup C$ and $claim(\varphi) = d$. An illustrative example of the reduction is given in Figure 15. By construction the formula φ is satisfiable iff there is a conflict-free set that attacks all arguments $c_i \in C$ iff there is an admissible set containing φ iff $\{c, d\}$ is a $cl\text{-}prf$ extension. We obtain that $Ver_{cl\text{-}prf}^{CAF}$ is NP-hard.

$\sigma \in \{cl\text{-}stg, cl\text{-}sem\}$: Let φ be an instance of 3-SAT, with φ given as a set of clauses $C = \{c_1, \dots, c_n\}$ over atoms in X, where negated atoms are denoted by $\bar{x}$. We construct $CAF_{\varphi} = (A, R, claim)$ with

$$\begin{aligned} A &= X \cup \bar{X} \cup C \cup \{y, z\} \\ R &= \{(x, \bar{x}), (\bar{x}, x) \mid x \in X\} \cup \{(l, c) \mid c \in C, l \in c\} \cup \{(c, c) \mid c \in C\} \cup \\ &\quad \{(x, y), (\bar{x}, y), (y, x), (y, \bar{x}) \mid x \in X\} \cup \{(z, z)\} \end{aligned}$$

with $claim(x) = claim(\bar{x}) = c$ for all $x \in X \cup \{z\}$, $claim(c_i) = d$ for all $c_i \in C$ and $claim(y) = d$. An illustrative example of the reduction is given in Figure 16. First notice that because of the specific use of symmetric attacks and the self attacks conflict-free sets and admissible sets coincide. Thus, also $cl\text{-}stg$ and $cl\text{-}sem$ coincide and it suffices to consider $cl\text{-}stg$ in the following. By construction the formula φ is satisfiable iff there is a conflict-free set that attacks all arguments $c_i \in C$ iff there is a cl-stage extension with range $\{c, d\}$ iff $\{d\}$ is not a cl-stage extension. We obtain that Ver_{σ}^{CAF} is NP-hard.

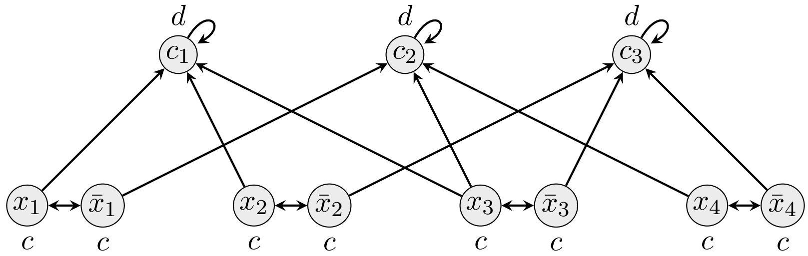


Figure 14: Construction from the proof of Proposition 47 for the formula φ given by the clauses $\{\{x_1, x_2, x_3\}, \{\bar{x}_1, x_3, x_4\}, \{\bar{x}_2, \bar{x}_3, \bar{x}_4\}\}$

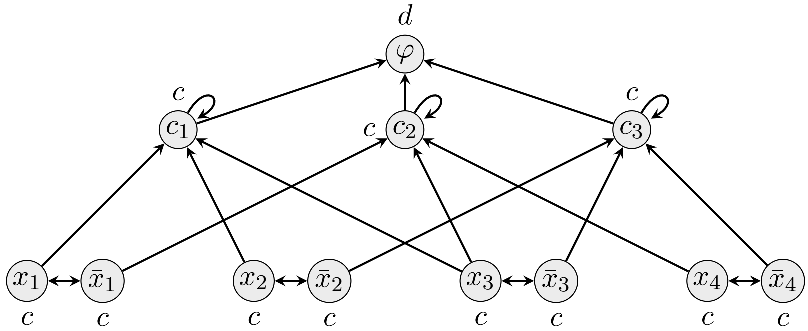


Figure 15: Construction from the proof of Proposition 47 for the formula φ given by the clauses $\{\{x_1, x_2, x_3\}, \{\bar{x}_1, x_3, x_4\}, \{\bar{x}_2, \bar{x}_3, \bar{x}_4\}\}$

Now consider the non-empty problems NE_σ^{CAF}. First, for the NP-hardness with $\sigma \in \{\textit{cl-stb}_{cf}, \textit{cl-stb}_{adm}\}$ consider the first reduction of this proof. By construction $\{c\}$ is the only candidate for being an extension and we already know that $\{c\}$ is an extension iff ϕ is satisfiable. Thus we obtain that there is a non-empty extension iff ϕ is satisfiable which shows NP-hardness.

For $\sigma \in \{\textit{cl-prf}, \textit{cl-stg}, \textit{cl-sem}\}$ we reuse the following construction from the proof of Proposition 46: For a given $CAF = (A, R, \textit{claim})$ with an arbitrary number of claims, create $CAF' = (A, R, \textit{claim}')$ with $\textit{claim}'(a) = c$ for all $a \in A$. Then $NE_\sigma^{CAF}(CAF) = NE_\sigma^{CAF}(CAF')$. $\square$

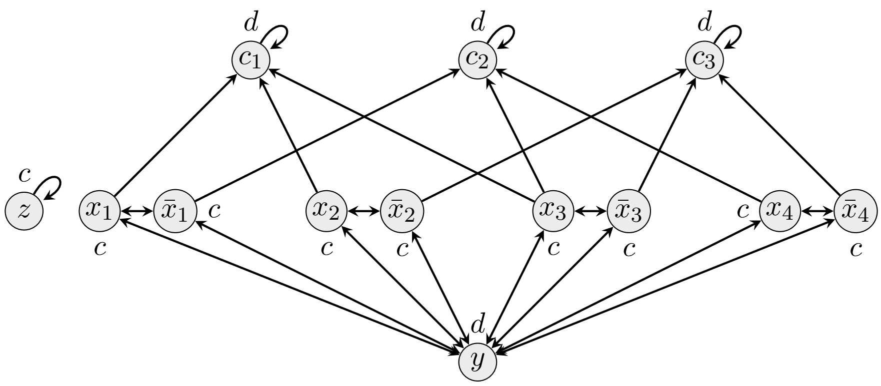


Figure 16: Construction from the proof of Proposition 47 for the formula φ given by the clauses $\{\{x_1, x_2, x_3\}, \{\bar{x}_1, x_3, x_4\}, \{\bar{x}_2, \bar{x}_3, \bar{x}_4\}\}$